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Suboptimal white noise estimators for discrete time systems with random delays

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ABSTRACT

This paper is concerned with the suboptimal deconvolution problems for discrete-time systems with random delayed observations and data losses. When the random delay is known online, i.e., time stamped, the random delayed system is reconstructed as an equivalent delay-free one by using measurement reorganization technique, and then a suboptimal input white-noise estimator with deterministic gains is developed under a new criteria. The estimator gain and its respective error covariance–matrix information are derived based on a new suboptimal state estimator. The obtained estimator is indeed a fixed-point smoother, based on which a fixed-lag white-noise smoother is derived. Further, it can be shown that the suboptimal input white-noise estimators converge to the steady-state ones under appropriate assumptions.

1. Introduction

The input white noise estimation or deconvolution estimation problem is an important research topic that has extensive applications in signal processing and telecommunications, such as seismic, radar, image and speech signal processing [1,2]. The purpose of the deconvolution problem is to estimate the unknown input signal of a system using the noise corrupted measurements, where the measurement is detected or transmitted via wireless sensor networks. As is well known that the packets in the transmission process usually suffer power and bandwidth constraints, then the occurrence of time delay and packet dropouts is unavoidable [3–6]. In this paper, we will study the deconvolution estimation problems for systems with random sensor delays and packet losses.

There exist several approaches to the deconvolution problem in the literature. Under the H_2 setting, Mendel et al. [1] presented an optimal input white-noise estimator with application to oil seismic exploration based on the Kalman filter. Sun [2] extended the result to the optimal information fusion white-noise filter and smoother. Deng et al. [7] presented a unified white-noise estimation theory including both input and measurement noise estimators. The estimators were developed by the modern time-series analysis method. In the case where the input is of colored noise, the optimal input estimation problems have also been received considerable attention [8,9]. The main approach applied there is the polynomial approach. With this approach, the deconvolution estimator is given in terms of spectral factorization and polynomial equations. Under the H_∞ setting, the optimal deconvolution has also been studied in [10–12]. Up to now, many results have been obtained for the deconvolution problems. However, the results mentioned above mainly focused on the delay-free systems.

There have been a vast number of solutions for the state estimation problem of system with random observation delays. In the case of observations transmitted to

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the estimator with irregular times, a recursive linear minimum variance state estimator was proposed via the state augmentation method [13]. For the situation that the random delay characterized by a set of distributed Bernoulli variables, the Kalman filter [14], and the robust H_∞ filters [15,16] have been developed. The rational of modeling the random delay as Bernoulli variable sequences has been justified in those papers. Alternatively, modeling the random delay as a finite state Markov chain is also a reasonable way. The relevant estimation results for this type of modeling can be found in [17], and the references therein. In a very recent study, the optimal filtering [18–21], the adaptive Kalman filtering [22], and the H_∞ filtering [23,24] problems associated with possible delay, uncertain observations and multiple packet dropouts are studied under a unified framework, respectively. So far, the obtained results mainly focus on the state estimation problems, but the white-noise estimation especially the stationary white-noise estimation for systems with random observation delays is seldom reported.

This paper investigates the suboptimal input white-noise estimation for the discrete-time systems with multiple random observation delays and packet dropouts. The key technique to be used for dealing with the random delay is the reorganization observation method, and by this method the random delayed system is transformed into a delay-free one. Then a suboptimal recursive input white-noise estimator is developed under a new performance index, in which the estimator gain and its corresponding error covariance matrix are deterministic and can be derived from a new suboptimal state estimator. Actually, the proposed estimator is a fixed-point smoother, which has the same dimension as the original signal. Further, a fixed-lag white-noise smoother can be derived from the fixed-point one directly. It can be shown that the suboptimal input white noise estimator converges to a steady-state white-noise estimator under natural assumptions, and thus a stationary white-noise smoother is obtained.

The remainder of this paper is organized as follows. In Section 2, we present the problem formulations. In Section 3, a suboptimal input white-noise estimator is presented by using the minimum mean squared error method, in which the estimator gains are deterministic. In Section 4, we show that the suboptimal input white-noise estimator converges to the steady-state one under the natural assumptions, and thus a stationary white-noise estimator is obtained. Finally, a numerical example is presented in Section 5, and the conclusions are drawn in Section 6 with some final comments.

2. Problem formulations

Consider the following discrete-time systems with random observation delays:

$$x(t+1)=\Phi x(t)+Gw(t), \quad x(0)=x_0, \tag{1}$$

$$y_{r(t)}(t)=Hx(t-r(t))+v(t-r(t)), \tag{2}$$

where $x(t)\in R^n$ is the state, $w(t)\in R^p$ is the input noise, $y_{\cdot}(t)\in R^q$ is the measurement and $v(t)\in R^q$ is the measurement noise. We assume that the initial state x_0, $w(t)$, and $v(t)$ are zero mean white noises with covariance matrices P_0, Q, and R, respectively, and x_0, $w(t)$ and $v(t)$ are mutually independent. The measurements in (2) are time-stamped, and the random delay $r(t)$ is bounded with $0\le r(t)\le r$ where r is given as the length of memory buffer, and its probability distributions are $\mathrm{Prob}(r(t)=i)=\rho_i$, $i=0,\dots,r$. If the measurements transmitted to the receiver with a delay larger than r, they will be considered as the ones lost completely. Thus the property $0\le\sum_{i=0}^{r}\rho_i\le 1$ is satisfied. Also $r(t)$ is independent of x_0, $w(t)$, and $v(t)$.

Under the above conditions, the possible received observations at each time t when $t>r$ are

$$y(t)=\mathrm{col}\{y_0(t),\dots,y_r(t)\}, \tag{3}$$

where

$$y_i(t)=\gamma_{i,t}Hx(t-i)+\gamma_{i,t}v(t-i), \quad i=0,\dots,r, \tag{4}$$

while $\gamma_{i,t}$ is a scalar binary function which indicates the arrival of the observation subject to the state $x(t-i)$, i.e.

$$\gamma_{i,t}=\begin{cases}1 & \text{If the observation for state } x(t-i) \text{ is received;}\\ 0 & \text{otherwise,}\end{cases}$$

and $\mathrm{Prob}(\gamma_{i,t}=1)=\rho_i(i=0,\dots,r)$. For cost saving in the control system design, the real state $x(t)$ is generally assumed to be observed at most one time, and thus $\gamma_{i,t}(i=0,\dots,r)$ must satisfy the following property:

$$\gamma_{i,t+i}\times\gamma_{j,t+j}=0, \quad i\neq j. \tag{5}$$

When $t<r$, the observation of (3) is written as

$$y(t)=\mathrm{col}\{y_0(t),\dots,y_t(t),0,\dots,0\}, \tag{6}$$

where we set $y_i(t)\equiv 0$ for $t<i\le r$.

The problem considered in this paper is that: given the observation sequences $\{\{y(s)\}_{s=0}^{t}\}$, find a minimum mean square error white-noise estimation $\hat{w}_e(t|t+N)$, such that

$$\mathrm{E}_{w,v,\gamma}\{[w(t)-\hat{w}_e(t|t+N)][w(t)-\hat{w}_e(t|t+N)]'\} \tag{7}$$

is minimum, while the estimation gain is deterministic. Note that $N=0$ is the filter, $N>0$ is the smoother, and $N<0$ is the predictor.

Remark 1. The performance index considered in (7) is different from that of the optimal white-noise estimation for delay-free systems [1], where the expectation is only taken over on the addition noise w and v. The estimator developed under the standard minimum variance error performance [1] is optimal since the information γ is available. However, the estimator gain is stochastic, and it is difficult to analyze the convergence of the estimator. The convergent property is an important performance index which is required in the estimation design. The expectation in (7) is taken over on the addition noise w, v, and the multiplicative noise γ simultaneously, and thus the obtained estimator is with deterministic gains, and the convergence can be guaranteed under appropriate assumptions. The precise definition to the above problem will be given below.

3. Suboptimal white-noise estimation

In this section, we will propose a new suboptimal input white-noise estimator with deterministic gains by