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A fast algorithm for nonconvex approaches to sparse recovery problems

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ABSTRACT

This paper addresses the problem of sparse signal recovery from a lower number of measurements than those requested by the classical compressed sensing theory. This problem is formalized as a constrained minimization problem, where the objective function is nonconvex and singular at the origin. Several algorithms have been recently proposed, which rely on iterative reweighting schemes, that produce better estimates at each new minimization step. Two such methods are iterative reweighted l_2 and l_1 minimization that have been shown to be effective and general, but very computationally demanding. The main contribution of this paper is the proposal of the algorithm WNFCS, where the reweighted schemes represent the core of a penalized approach to the solution of the constrained nonconvex minimization problem. The algorithm is fast, and succeeds in exactly recovering a sparse signal from a smaller number of measurements than the l_1 minimization and in a shorter time. WNFCS is very general, since it represents an algorithmic framework that can easily be adapted to different reweighting strategies and nonconvex objective functions. Several numerical experiments and comparisons with some of the most recent nonconvex minimization algorithms confirm the capabilities of the proposed algorithm.

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1. Introduction

The problem of recovering a sparse signal from a very low number of linear measurements arises in many real application fields, ranging from error correction and lost data recovery, to image acquisition and reconstruction.

According to the recent compressed sensing theory, this problem can be formalized as a constrained minimization problem, where the objective function induces sparsity in the solution. More precisely, let $x \in \mathbb{R}^N$ be an unknown vector, whose nonzero components are only K, with $K \ll N$ (such a vector will be called *K-sparse* vector). The sparse recovery problem can be cast as: given the $M \times N$ measurement matrix Φ, with $M < N$, find the *K-sparse* vector $x \in R^N$ from its measurements $y = \Phi x$, by solving

$$\min_{u \in \mathbb{R}^N} F(u) \quad \text{subject to } \Phi u = y, \tag{1}$$

where the sparsity inducing function $F(u)$ allows us to select, among the infinitely many solutions of the underdetermined linear system $\Phi u = y$, the desired one.

The most natural choice for $F(u)$ is given by $F(u) = \|u\|_0$, since the l_0-norm counts the number of nonzero entries of u. The solution of

$$\min_{u \in \mathbb{R}^N} \|u\|_0 \quad \text{subject to } \Phi u = y, \tag{2}$$

can be evaluated by a combinatorial search over all possible K-sparse u vectors satisfying $\Phi u = y$. Even if the true signal could be recovered from $M \geq 2K$ random

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measurements, the cost of the combinatorial search is prohibitive and other strategies must be considered, which relax the l_0-norm minimization into other more tractable sparsity inducing functions. A common approach is to use the l_1-norm as a convex relaxation of the l_0-norm, and to solve the convex problem

$$\min_{u \in \mathbb{R}^N} \|u\|_1 \quad \text{subject to } \Phi u = y. \tag{3}$$

It has been shown in [4,6] that the above problem (Eq. (3)) is equivalent to that of Eq. (2) provided that

- every set of K columns of the measurement matrix Φ approximatively forms an orthogonal system (restricted isometry property);
- the measurement number satisfies

$$M = O\left(K \log\left(\frac{N}{K}\right)\right). \tag{4}$$

(this bound applies to random Gaussian matrices Φ, for other bounds see [7,5]).

For a more detailed description of the restricted isometry property and other bounds on the acquisition matrix, the interested reader is referred to [6,7].

Many algorithms have been developed to efficiently solve the problem in Eq. (2) (see [2,3,14,17,20,21,25,26] and the references therein), but recent results [8,10,27,22,23] have shown that the use of different sparsity inducing functions $F(u)$ allows us to exactly and efficiently recover a sparse signal from a lower number of measurements than the l_1 norm.

In particular, we refer to the nonconvex sparsity inducing functions, proposed in the most recent literature, which seem to better reproduce the sparsifying action of the l_0-norm than the l_1. Among them, we mention the l_q-norm, for $0<q<1$ [9,10,22], and the *log-sum* and *atan* sparsity inducing functions [8,24]. To solve the corresponding nonconvex minimization problems, numerical strategies have been proposed that iteratively minimize a convex majorization of the nonconvex objective function. This technique gives rise to the iterative reweighted algorithms, whose experimental results assess the improved performance of the nonconvex approach with respect to that of l_1 minimization, but at the expense of an increase in the computing time.

The contribution of this paper to the nonconvex approach is twofold: (a) we propose a novel nonconvex sparsity inducing function which represents a continuous relaxation of the l_0-norm and whose shape can be modeled via suitably adjusting a positive parameter; (b) to solve the corresponding nonconvex constrained minimization problem displayed in Eq. (1) we propose a fast iterative algorithm, that integrates both the strategy to overcome the nonconvexity of the objective function and the adaptive adjustment of the sparsity inducing function in a penalization approach. The proposed weighted nonlinear filters for compressed sensing (WNFCS) algorithm represents a general framework, which can successfully be used with most of the nonconvex sparsity inducing functions proposed in the literature. Its convergence to a local minimum of the original nonconvex objective function is guaranteed by the proof that the descent properties both of the majorization–minimization (MM) approach and of the penalization method are still valid in the new context. WNFCS is fast, since it is much less computationally demanding than the classical reweighted approaches, and succeeds in exactly recovering a sparse signal from a lower number of measurements than the l_1 minimization. Moreover, when the available data reach the amount necessary to the success of the l_1 minimization, remarkably fewer iterations are required by WNFCS, than those used by l_1 minimization to obtain comparable reconstruction results. Numerical experiments and comparisons with some of the most recent nonconvex minimization algorithms assess the capabilities of the proposed algorithm.

This paper is organized as follows. In Section 2 we recall some nonconvex sparsity inducing functions presented in the literature and we propose a new one. The classical reweighted approaches to nonconvex minimization are briefly mentioned in Section 3, along with their strengths and weaknesses. In Section 4 we propose to use a reweighting scheme as the core of an iterative penalized procedure, and we analyze its convergence properties. Section 5 describes the main structure of the WNFCS algorithm and proposes some improvements that enhance its efficiency. A large set of numerical experiments is presented in Section 6, confirming the effectiveness of the proposed strategy. Section 7 closes the work with some brief conclusions.

2. Nonconvex sparsity inducing functions

The problem of finding different sparsity inducing alternatives to l_0 or l_1 minimization has been addressed in the literature mainly by preserving the separability properties of both the l_0 and l_1 norms, thus considering objective functions that can be expressed as

$$F(u) = \sum_{i=1}^{N} \psi(|u_i|) \quad u \in \mathbb{R}^N. \tag{5}$$

The function $\psi : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is required to provide a function $F(u)$ that reproduces at best the sparsifying action of the l_0 norm, while still maintaining some good properties of the l_1 norm, such as continuity and differentiability (for $u \neq 0$).

Since it has been experimentally shown that the nonconvex relaxation of the l_0 norm outperforms the l_1 minimization, several concave sparsity inducing functions, more or less closely resembling the l_0 norm, have been proposed in the recent literature for sparsity recovery. Among them, we mention the l_q function, with $0<q<1$, proposed in [9] as a smooth relaxation of the l_0, the *log-sum* sparsity inducing function, successfully experimented in [8] for several sparse recovery problems, and the *atan* function, recently experimented in [16]. All these sparsity inducing functions are displayed in Table 1, together with their first derivatives, and are represented in Fig. 1(a)–(c), for different values of the parameter ϵ for *log-sum* and *atan*, and q for l_q. All of them are concave,

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