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# The neural circuits for arithmetic principles

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A B S T R A C T

Arithmetic principles are the regularities underlying arithmetic computation. Little is known about how the brain supports the processing of arithmetic principles. The current fMRI study examined neural activation and functional connectivity during the processing of verbalized arithmetic principles, as compared to numerical computation and general language processing. As expected, arithmetic principles elicited stronger activation in bilateral horizontal intraparietal sulcus and right supramarginal gyrus than did language processing, and stronger activation in left middle temporal lobe and left orbital part of inferior frontal gyrus than did computation. In contrast, computation elicited greater activation in bilateral horizontal intraparietal sulcus (extending to posterior superior parietal lobule) than did either arithmetic principles or language processing. Functional connectivity analysis with the psychophysiological interaction approach (PPI) showed that left temporal-parietal (MTG-HIPS) connectivity was stronger during the processing of arithmetic principle and language than during computation, whereas parietal-occipital connectivities were stronger during computation than during the processing of arithmetic principles and language. Additionally, the left fronto-parietal (orbital IFG-HIPS) connectivity was stronger during the processing of arithmetic principles than during computation. The results suggest that verbalized arithmetic principles engage a neural network that overlaps but is distinct from the networks for computation and language processing.

## 1. Introduction

Arithmetic calculation has three major cognitive components: conceptual knowledge, arithmetic procedural knowledge, and arithmetic facts (Sokol and McCloskey, 1991). The core of arithmetic conceptual knowledge is arithmetic principles, which are the fundamental laws or regularities underlying arithmetic (Prather and Alibali, 2009). Examples of arithmetic laws include the commutative law (3+2=2+3, or 3×2=2×3) and the associative law (e.g., 2×3+3×3=(2+3)×3). Other arithmetic principles include the inverse relation of operations (e.g., 3+4-4=3, 3×4÷4=3), 0- or 1-based computation (e.g., n+0=n, n×1=n, n÷1=n).

Arithmetic principles have been extensively investigated in behavioral studies (e.g., Canobi, 2005; Prather et al., 2009; Rasmussen et al., 2003; Robinson et al., 2006). Researchers have found that even preschoolers can understand and apply arithmetic principles (e.g., Klein and Bisanz, 2000; Rasmussen et al., 2003; Vilette, 2002). For example, Klein et al. (2000) used a nonverbal procedure to present both inversion (e.g., 3+4−4) and standard problems (e.g., 3+5−4) to 4-year-olds. It was found that solutions were faster for inversion than for standard problems. Similar evidence was found among 3-year-old children (Sherman and Bisanz, 2007). However, older children are more likely than younger children apply arithmetic principles to solving arithmetic problems (Canobi, 2005; Robinson et al., 2006). For example, Robinson et al. (2006) reported that the inversion strategy was used significantly more often in grade 8 than in grade 6 when solving addition/subtraction inversion problems and multiplication/division inversion problems. Canobi (2005) also found that when solving computation problems, the percentages of 5- to 7-year-old children who use the inversion strategy increased with age.

A number of behavioral studies have found that participants' knowledge of arithmetic principles is not associated with their performance on computation problems (e.g., Bryant et al., 1999; Rasmussen et al., 2003; Sherman et al., 2007). For example, Rasmussen et al. (2003) found that children's ability to add 9's was not related to their use of the inversion principle for problems involving "+9 − 9". One study (Canobi et al., 1998) nonetheless found that the use of relational properties in computation such as additive composition, commutativ-

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ity, and associativity principles was related to speed and accuracy in solving unrelated problems. These results suggest that the understanding of arithmetic principles and the computational arithmetic ability are two related but independent cognitive components. Consistent with this perspective, neuropsychological studies have shown that these two components can be selectively impaired (e.g., Cappelletti et al., 2001, 2005; Dehaene & Cohen, 1997; Hittmair-Delazer et al., 1994; 1995; McCloskey et al., 1991; Pesenti et al., 2000; Sokol et al., 1991; Warrington, 1982). Specifically, simple computation is impaired but the understanding of arithmetic principles is not, when there are damages to brain regions such as the basal ganglia (patient BE, Hittmair-Delazer et al., 1994), left parietal-occipital cortex (patient DRC, Warrington, 1982), left temporal lobe (patient IH, Cappelletti et al., 2001, 2005), entire white matter (patient DA, Hittmair-Delazer et al., 1995), bilateral temporoparietal areas (patient DF, Pesenti et al., 2000), or right inferior parietal lobule (patient MAR, Deheane & Cohen, 1997). For example, Hittmair-Delazer et al. (1994) reported having a stroke affecting left basal ganglia, patient BE showed impaired simple computation (e.g., 18÷6, 4×9) but could apply arithmetic principles to derive correct answers (e.g., 4×9=9×2+9×2=36).

Researchers have also reported cases for which the processing of arithmetic principles was selectively impaired but arithmetic computation was relatively intact (Delazer and Benke 1997; Sokol et al., 1991). After the surgical removal of a left parietal tumor, Patient JG (Delazer et al., 1997) was reported to have completely lost her arithmetic conceptual knowledge, including basic concepts of the four operations and arithmetic principles (i.e., commutativity law, inverse principle relation), but preserved some ability to solve simple computation problems (multiplications and some additions and subtractions). The patient was unable to answer questions such as "If 13+9 is 22, what is 9+13?", "if 13+9=22, what is 22−9?", which required the application of the commutative law and inverse principle, respectively. After suffering from left frontal contusion, Patient GE also showed selective impairment in solving arithmetic problems involving 0 (0-based computational rule) (Sokol et al., 1991). Specifically, for the 0×n problems, he was 0% correct (0/390), but for problems with two non-0 operands, his error rate was 8.8% (156/1763). These studies suggest that the focal brain lesions in left parietal cortex and left frontal cortex can lead to impairment of the understanding of arithmetic principles.

Although the neuropsychological studies reviewed above showed that arithmetic principles' processing can be dissociated from numerical processing, these studies lacked spatial resolution to pinpoint the neural basis of arithmetic principles. Thus far, there has been only one neuroimaging study of arithmetic principles (Jost et al., 2009). Jost et al. (2009) investigated the neural activation of 0-based problems in multiplication and found that the 0-based multiplication problems solved by rule application elicited greater activation at left caudate nucleus, right inferior frontal gyrus, bilateral middle temporal gyrus, left angular gyrus, and right cuneus extending to the precuneus than those solved by fact retrieval (e.g., 7×8). The current study extended Jost et al. work by including arithmetic principles beyond the 0-based rule in multiplication.

To understand the neural basis of the processing of arithmetic principles, we also need to dissociate it from the processing of general semantic knowledge (e.g., Cappelletti et al., 2012; Julien et al., 2008). Several neuropsychological studies reported dissociation between arithmetic principles and general semantic knowledge (Cappelletti et al., 2005, 2012; Julien et al., 2008; Julien et al., 2010; Sokol et al., 1991; Zamarian et al., 2006). For example, semantic dementia patient IH was reported to have with well-preserved arithmetic conceptual knowledge (including arithmetic principles and operations), but failed in general semantic tasks such as picture naming and word classification (Cappelletti et al., 2005). Semantic dementia patient SG performed well in addition/multiplication arithmetic principles, as well as definitions of operation tasks, but was partially impaired in a comprehensive test of verbal semantic knowledge assessing living and non-living categories (providing 98 incorrect answers out of 480 questions) (Zamarian et al., 2006). Nevertheless, there exists evidence suggesting that arithmetic conceptual knowledge is not totally separated from general semantic knowledge (Cheng et al., 2013; Julien et al., 2008; Julien et al., 2010). For instance, SD patients made procedural errors in a multi-digit calculation task, which suggested a progressive degradation in conceptual understanding of arithmetic (Julien et al., 2008). Patients with severe semantic dementia showed more impairment in judging quantifiers' (e.g., "many", "none") semantic relatedness than the patients with mild semantic dementia, which indicated that quantifier processing is associated with general semantic processing and can be impaired due to temporal lobe damage (Cheng et al., 2013). These observations suggested that the temporal lobes might play an important role in arithmetic conceptual knowledge.

The goal of the current neuroimaging study was to investigate how different brain regions jointly subserve the processing of arithmetic principles as compared to numerical computation and general language processing. Two hypotheses were tested. The first hypothesis is that arithmetic principles involve visualization (or mental models) and hence should activate the bilateral horizontal segments of the intraparietal sulcus (IPS). The mental models integrate the relations of mathematical concepts involved in arithmetic principles. They involve mental imageries of mathematical expressions with spatial information (e.g., "Exchanging the position of operands in addition does not change the result", "For division, the position of dividend and divider should not be exchanged"). The processing of such spatial information should activate the IPS (e.g., Boccia et al., 2014; Moore and Armstrong, 2003; Szczepanski et al., 2010; Wolbers and Hegarty, 2010).

The second hypothesis is that arithmetic principles are a type of conceptual knowledge and are hence processed in the semantic information processing areas including the left middle temporal gyrus (MTG) and left prefrontal cortex. Left MTG is an important semantic hub (e.g., Binder et al., 2009; Wu et al., 2012; Kuperberg et al., 2008). It has been related to mathematical concept processing and quantity processing (Wei et al., 2014; Zhang et al., 2012). Furthermore, damage to left temporal lobe was associated with progressive degradation in conceptual understanding of arithmetic (Julien et al., 2008; Julien et al., 2010). The orbital part of the IFG has also been reported to be responsible for semantic processing (e.g., Kuperberg et al., 2008; Wagner et al., 2001) and specific mathematical semantic processing (Zhang et al., 2012; Wei et al., 2014). For example, Wagner et al. (2001) found that the orbital part of left IFG was involved in controlled semantic retrieval. Finally, left frontal lesion has been linked to impairment in the understanding of arithmetic principles (Delazer and Butterworth, 1997; Sokol et al., 1991).

In the current study, we used sentences rather than symbols to describe arithmetic principles in order to match the format of general semantic processing. For example, the law of additive communativity was expressed as "Exchanging the position of two operands in addition does not change their sum", rather than its symbolic expression of "a +b=b+a". A verification paradigm was used for all three tasks. Participants were asked whether a particular statement was correct or incorrect. To match the verbal processing involved in arithmetic principles, the numerical computation verification task was also presented in verbal context (e.g., "When number 8 is first divided by number 4, then multiplied by number 3, the final result is number 12"). For the general language processing task, participants were asked to judge whether descriptions of everyday life were true or not (e.g., "When school starts, new students come one after another and register").

## 2. Materials and methods

### 2.1. Participants

Thirty right-handed undergraduates (15 male; aged 19.1–24.6