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H_∞ filtering of network-based systems with random delay

Hongbo Song, Li Yu *, Wen-An Zhang

Department of Automation, Zhejiang University of Technology, Hangzhou 310032, PR China

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The H_∞ filtering problem is studied for a class of network-based systems with random delay in discrete-time domain. A new model is proposed to describe the filtering system with random sensor-filter delay which may be longer than one sampling period. The random delay is modeled as a Markov chain and the resulting filtering error system is a Markovian switched system with random state delay. By using a properly constructed Lyapunov function and the state transform technique, sufficient conditions for the existence of the H_∞ filters are presented in terms of linear matrix inequalities. An optimization problem with LMIs constraints is formulated to design the H_∞ filter which guarantees that the filtering error system is mean-square exponentially stable with a prescribed decay rate and ensures an optimal H_∞ disturbance attenuation level. An illustrative example is given to demonstrate the effectiveness of the proposed results.

1. Introduction

In many modern complex and distributed control systems, remotely located sensors, actuators, controllers and filters are often connected over a sharing communication network. Systems with such architectures are often called the network-based systems, which bring a lot of advantages such as low cost, simple installation and maintenance, increased system agility and so on [1–3]. In spite of these advantages, the sharing network makes the analysis and synthesis of such network-based systems challenging. Recently, the network-based control system, which is called the networked control system (NCS), has attracted much research interest [1–4]. On the other hand, signal estimation over network is important in many applications such as remote sensing, space exploration, and sensor networks. Therefore, the network-based signal estimation is also a potential researching field which needs to be fully investigated [4,17].

It is known that when signals are transmitted through the network, the time delay or even packet dropout is unavoidable between the senders and the receivers. Moreover, the network-induced delays and packet dropouts are often random so that many classical filtering schemes such as Kalman filtering should be modified to solve the filtering problem over network. The Kalman filtering problem under random delay effect and packet dropout effect was studied in [5,6] and in [7–11], respectively. However, one primary limitation of Kalman filtering is that the external disturbances are required to be stationary Gaussian noises with known statistic property. Since such a requirement is not always satisfied in practical applications, some alternatives such as the H_∞ and the H_2 filtering schemes are introduced to study the signal estimation problem [12,15], and they are also applied in the filtering problem for network-based systems. The optimal H_2 and the optimal H_∞ filtering problem for network-based system with packet dropout were studied in [15] and in [18], respectively. As for the network-induced delay issue, the H_∞ filtering and the robust H_∞ filtering problem for network-based system with short random delay were studied in [13] and in [14], respectively. In [15], the optimal H_2 filtering problem with

* Corresponding author. Tel/fax: +86 571 88320287.
E-mail addresses: di7gan_shb@hotmail.com (H.B. Song), lyu@zjut.edu.cn (L. Yu).

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random one step sensor delay, multiple packet dropout and uncertain observation was studied in a unified framework. In [16], the robust filtering problem was investigated for network-based systems with randomly varying sensor delay and variance constraints. It should be pointed out that the random delay is modeled as a Bernoulli distributed sequence and the technique of transforming the random delay into a stochastic parameter in the system representation is used in [13–16]. However, the technique can only handle one step delay. It is also an important observation that when the random delay is longer than one sampling period, the modeling of filtering error system will be difficult because various plant outputs may arrive at the filter during one sampling period and new data may arrive at the filter before the old one, which make the modeling and filtering of network-based system with delay longer than one sampling period challenging. Up to date, few results have been reported in the existing literature for such topic. In [17], the H_∞ filtering problem for network-based system with time-varying long delay was studied in continuous-time domain. However, the considered delay is not random. To the best of the authors' knowledge, the problem of network-based H_∞ filtering with randomly varying long delay has not yet been investigated, which motivates the present research.

In this paper, the H_∞ filtering problem is investigated for a class of network-based systems with random delay that may be longer than one sampling period. First, a logical relation between the delays of the current time step, the previous time steps and filter input delay is explicitly established. The random delay is modeled as a Markov chain. Then by the relation we present, the overall filtering error system is finally modeled as a Markovian switched time-delay system whose subsystems may depend on the delays of the current time step or some previous time steps. By applying the Lyapunov method and the state transformation technique, sufficient criteria are derived in terms of LMIs (linear matrix inequalities) for the resulting filtering error system to be exponentially stable and to have a prescribed H_∞ performance. An optimization problem with LMIs constraints is formulated to design the desired filter which minimizes the H_∞ performance level for the filtering error system. An illustrative example is finally provided to demonstrate the effectiveness of the proposed results.

2. Modeling of the filtering error system

The structure of the considered network-based filtering system is shown in Fig. 1, and the plant is described by the following discrete-time linear time-invariant system model:

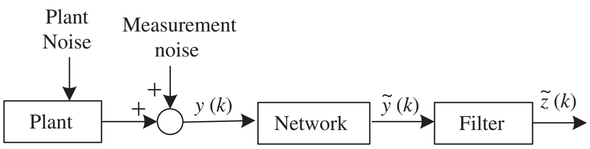

Fig. 1. Structure of the network-based filter.

$$\begin{cases} x(k+1) = Ax(k) + Bw(k), \\ y(k) = Cx(k) + Dw(k), \\ z(k) = Lx(k), \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the system state, $w(k) \in R^m$ is the disturbance input and belongs to $L_2[0,\infty)$, $y(k) \in R^p$ is the measured output, and $z(k) \in R^r$ is the signal to be estimated. A, B, C, D and L are constant matrices of appropriate dimensions. We consider the full order linear filters of the following form:

$$\begin{cases} x_f(k+1) = A_f x_f(k) + B_f \tilde{y}(k), \\ \hat{z}(k) = C_f x_f(k) + D_f \tilde{y}(k), \end{cases} \tag{2}$$

where $x_f(k) \in R^n$ is the filter state, $\tilde{y}(k) \in R^p$ is the filter input, and $\hat{z}(k) \in R^r$ is the estimated signal, A_f, B_f, C_f and D_f are filter parameter matrices to be determined.

We use $\rho(k)$ to denote the random sensor-filter delay and it is assumed to be bounded, that is, $0 \leqslant \rho(k) \leqslant N$, where N is a known integer. We model $\rho(k)$ as a Markov chain that takes values in $\mu_\rho = \{0,1,\ldots,N\}$ with given transition probability matrix $\Gamma = [\rho_{ij}]$. The transition probability ρ_{ij} is defined as follows:

$$\rho_{ij} = \text{Prob}\{\rho(k+1) = j | \rho(k) = i\}, \tag{3}$$

where $\rho_{ij} \geqslant 0$, $\sum_{j=0}^{N} \rho_{ij} = 1$ for all $i,j \in \mu_\rho$, Prob$\{\bullet\}$ stands for the occurrence probability of an event. The number of the state of the Markov chain and the elements of the transition probability matrix can be obtained, respectively, via testing the delays in the network-based filtering system and analyzing the statistic characteristic of the delays by applying certain algorithms such as that in [19]. The method of modeling the random delay as a Markov chain was introduced in [3], in which the stabilization problem of NCSs with random delays was studied. Since the network-induced delay may be longer than one sampling period, several measured outputs may arrive at the filter side over one sampling period. It is assumed that the filter always uses the most recent measured output available at the filter side to update its input, and that if no measured output arrives at the filter over one sampling period, the filter input will hold at its previous value. By the above analysis and assumption, it can be seen that the filter input $\tilde{y}(k)$ may take values in $\{y(k), y(k-1), \ldots, y(k-N)\}$ at each sampling instant, which will result in N+1 different system dynamics of the filtering error system. For example, when $\tilde{y}(k) = y(k)$, by augmenting the state variable as $\tilde{x}(k) = [x^{\mathrm{T}}(k),\ x_f^{\mathrm{T}}(k)]^{\mathrm{T}}$, the disturbance input as $\tilde{w}(k) = [w^{\mathrm{T}}(k)\ w^{\mathrm{T}}(k-1)\ \cdots\ w^{\mathrm{T}}(k-N)]^{\mathrm{T}}$, and defining $e(k) = z(k) - \hat{z}(k)$, we obtain the following filtering error system, which is one subsystem of the overall filtering error system,

$$\begin{cases} \tilde{x}(k+1) = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} B & 0 & \cdots & 0 \\ B_f D & 0 & \cdots & 0 \end{bmatrix} \tilde{w}(k), \\ e(k) = [L - D_f C\ -C_f]\tilde{x}(k) + [-D_f D\, 0\ \cdots\ 0]\tilde{w}(k). \end{cases} \tag{4}$$