Signal Processing 89 (2009) 283–290

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Blind paraunitary equalization

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ARTICLE INFO

Article history:
Received 26 October 2007
Received in revised form
2 June 2008
Accepted 28 August 2008
Available online 1 October 2008

Keywords:
Blind deconvolution
Blind equalization
Multiple-input/multiple-output (MIMO)
High order statistics (HOS)
Paraunitary filter
Pairwise cumulants

ABSTRACT

In this paper a blind multiple-input/multiple-output (MIMO) space–time equalizer is described, dedicated to convolutive mixtures when observations have been pre-whitened. Filters preserving space–time whiteness are paraunitary; a parameterization of such filters with plane rotations is proposed. Theoretical developments then lead to a numerical algorithm that sweeps all pairs of delayed outputs. This algorithm involves the solution of a polynomial system in two unknowns, whose coefficients depend on the output cumulants. Simulations and performance of the numerical algorithm are reported.

1. Introduction

In actual digital communication systems, the equalization problem is solved using learning sequences. These sequences, known by transmitter and receiver, permit to estimate channel parameters. Nevertheless, learning sequences may be seen as reducing the throughput, because they occupy a non-negligible space in transmitted sequences. In other words, less actual information is transmitted, i.e. the useful data rate is lower than the system data rate. Next, in some situations, e.g. interception in electronic warfare, learning sequences exist but are not known to the receiver.

Blind separation methods for multiple-input multiple-output (MIMO) channels have raised an increasing interest for digital communications since they do not need learning sequences. Most blind MIMO equalization techniques use high order statistics (HOS) for separating signals [1–8]. This can be implicit through constant modulus [9–12] or constant power [13] criteria. Purely deterministic approaches also exist and exploit either the finite alphabet property [14–16] or the presence of another diversity in addition to time and space [17,18].

Our main contribution consists of a block algorithm dedicated to blind MIMO equalization and exploiting HOS of observations [19]. The particularity of our method is that it is based on a factorization of paraunitary filters [20], as suggested in [21,22]. The paraunitary condition is not very restrictive since prewhitening can always be performed in a first stage (in a non-unique manner) even if it is not always an obvious operation [22–25]. Moreover, the algorithm designed herein can be implemented either "on-line" or "off-line".

More precisely, our algorithm can run iterations on a single given block until convergence, or on the contrary, run a new iteration after each symbol arrival. Convergence of on-line algorithms is known to be much longer in terms of number of symbols required (typically several thousands of symbols).

Note that specific criteria [26,27] and algorithms [28] dedicated to paraunitary channels have already been proposed. Unfortunately, the paraunitary constraint is not accurately verified in the modeling of [27,28],

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doi:10.1016/j.sigpro.2008.08.014

especially at low signal-to-noise ratios (SNR), because the channel is parameterized with a semi-unitary matrix.

The paper is organized as follows. In the second section, model and hypotheses are presented. The parameterization of MIMO paraunitary filters is introduced in Section 3, and input–output cumulant relations are established. A contrast criterion is proposed in Section 4, for finding equalizer parameters and separating source signals. An iterative algorithm built from previous theoretical results is described in the fifth section. The core of the algorithm is a Jacobi-type iteration, in which all possible Givens rotations in the parameterization of the paraunitary filter are swept until convergence. Finally, the performance for various SNR is illustrated in the last section.

2. Model and notations

Throughout the paper, (T) stands for transposition, (H) for conjugate transposition, $(*)$ for complex conjugation, and $\jmath = \sqrt{-1}$. Also denote by $\mathbb{Z}$ the set of integers, by $\mathbb{N}$ the subset of positive integers and by z^{-1} the time-delay operator. Vectors and matrices are denoted with bold lowercase and bold uppercase letters, respectively. The entries of matrix $\boldsymbol{G}$ are denoted G_{ij}, where subscript ij denotes the i-th row and the j-th column of $\boldsymbol{G}$.

Let us consider a digital communication system of N antennas and N receivers in a multipath environment. Let $\boldsymbol{s}(n) = (s_1(n), \ldots, s_N(n))^{\mathsf{T}}$ denote the N-dimensional source vector of baseband complex signals at time n and $\boldsymbol{w}(n) = (w_1(n), \ldots, w_N(n))^{\mathsf{T}}$ the N-dimensional observation vector. Let $\boldsymbol{C}[z]$ be the transfer function of the linear time invariant (LTI) mixing channel and $\{\boldsymbol{C}(k), k = 0, \ldots, K\}$ be the sequence of $N \times N$ matrices of the complex finite impulse response (FIR) of the channel. We then have

$$\boldsymbol{w}(n) = \sum_{k=0}^{K} \boldsymbol{C}(k)\boldsymbol{s}(n-k), \tag{1}$$

where K denotes the memory length of the channel. Also denote

$$\boldsymbol{C}[z] = \sum_{m=0}^{K} \boldsymbol{C}(m) z^{-m}. \tag{2}$$

The case of instantaneous mixtures is not considered in the present paper, since already addressed via pairwise processing by various authors since 1991. Thus, it will be assumed that $K > 0$.

The multichannel blind deconvolution problem consists of finding an LTI filter $\boldsymbol{H}[z]$, i.e. the *equalizer*, in order to retrieve the N input signals $s_i(n), i \in \{1, \ldots, N\}, \forall n \in \mathbb{Z}$, solely from the observation of the outputs $\boldsymbol{w}(n)$ of the unknown LTI channel $\boldsymbol{C}[z]$.

Let $\hat{\boldsymbol{s}}(n) = (\hat{s}_1(n), \ldots, \hat{s}_N(n))^{\mathsf{T}}$ be the N-dimensional estimated source vector. This means, with the above notation:

$$\hat{\boldsymbol{s}}(n) = \sum_{l=0}^{L} \boldsymbol{H}(l)\boldsymbol{w}(n-l), \tag{3}$$

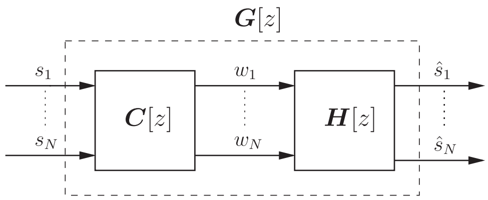

Fig. 1. Global system $\boldsymbol{G}$: sources s_i are filtered by channel $\boldsymbol{C}[z]$ and observations w_i are equalized by $\boldsymbol{H}[z]$.

where L is the memory length of the equalizer, or, as a function of original sources themselves:

$$\hat{\boldsymbol{s}}(n) = \sum_{m=0}^{K+L} \boldsymbol{G}(m)\boldsymbol{s}(n-m), \tag{4}$$

where $\boldsymbol{G}[z]$ denotes the global filter $\boldsymbol{G}[z] = \boldsymbol{H}[z]\boldsymbol{C}[z]$ (see Fig. 1 above).

Definition 1 (*Paraunitarity*). A square polynomial matrix $\boldsymbol{H}[z] \in \mathbb{C}^{N \times N}$ is said to be paraunitary [21] if

$$\boldsymbol{H}^{\mathsf{H}}[1/z^*]\boldsymbol{H}[z] = \boldsymbol{I}_N = \boldsymbol{H}[z]\boldsymbol{H}^{\mathsf{H}}[1/z^*], \tag{5}$$

where $\boldsymbol{I}_N \in \mathbb{R}^{N \times N}$ is the identity matrix.

In this paper, we assume the following hypotheses:

(H1) Inputs $s_i(n), \forall i \in \{1, \ldots, N\}, \forall n \in \mathbb{Z}$, are mutually independent and identically distributed (i.i.d.) zero-mean random processes, with unit variance.

(H2) Vector $\boldsymbol{s}(n)$ is stationary up to the considered order r, $r \geqslant 3$, i.e. $\forall i \in \{1, \ldots, N\}$, the order-$r$ marginal cumulants,

$$\mathrm{C}_p^q[s_i] = \mathrm{Cum}[\underbrace{s_i(n), \ldots, s_i(n)}_{p \text{ terms}}, \underbrace{s_i^*(n), \ldots, s_i^*(n)}_{q=r-p \text{ terms}}]$$

do not depend on n. For definitions of cumulants, refer to [29] and references therein.

(H3) At most one source has a zero marginal cumulant of order r.

(H4) $\boldsymbol{C}[z]$, $\boldsymbol{H}[z]$, and hence $\boldsymbol{G}[z] = \boldsymbol{H}[z]\boldsymbol{C}[z]$ are all paraunitary. Thus we have the global relation:

$$\boldsymbol{H}[z]\boldsymbol{C}[z]\boldsymbol{C}^{\mathsf{H}}[1/z^*]\boldsymbol{H}^{\mathsf{H}}[1/z^*] = \boldsymbol{I}_N. \tag{6}$$

Remark 2. One can always whiten the observations by using a filter that factorizes the second-order power spectrum, i.e. a classical prewhitening of the observations [22,23]. In other words, paraunitary filters are relevant after a space–time standardization of observations (second-order white with unit covariance). The way space–time whitening is implemented is out of the scope of this paper, and it is assumed that observations $\boldsymbol{w}(n)$ are space–time white.

As is now well known, statistical independence of $\hat{s}_i$ can only allow to blindly recover the sources up to a permutation matrix $\boldsymbol{P}$, and a diagonal delay filter $\boldsymbol{\Lambda}[z]$, so that $\boldsymbol{H}[z]\boldsymbol{C}[z] = \boldsymbol{\Lambda}[z]\boldsymbol{P}$.