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Review

A general framework for second-order blind separation of stationary colored sources

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ABSTRACT

This paper focuses on the blind separation of stationary colored sources using the second-order statistics (SOS) of their instantaneous mixtures. We first start by presenting a brief overview of existing contributions in that field. Then, we present necessary and sufficient conditions for the identifiability and partial identifiability using a finite set of correlation matrices. These conditions depend on the autocorrelation function of the unknown sources. However, it is shown here that they can be tested directly from the observation through the decorrelator output. This issue is of prime importance to decide whether the sources have been well separated. If that is not the case then, further treatments will be needed. We then propose an identifiability testing based on resampling (jackknife) technique that is validated by simulation results. Secondly, we present an iterative blind source separation method using SOS and natural gradient technique. This algorithm has a number of attractive properties including its simplicity and "easy" generalization to adaptive or convolutive schemes. Asymptotic performance analysis of this method is performed. Several numerical simulations are presented, to assess the theoretical results w.r.t. the "separability" testing, to demonstrate the effectiveness of the gradient-type decorrelation method and to validate the theoretical expression of the asymptotic performance index.

Contents

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1. Introduction

Source separation aims to recover multiple sources from multiple observations (mixtures) received by a set of linear sensors. The problem is said to be "blind" when the observations have been linearly mixed by the transfer medium, while having no *a priori* knowledge of the transfer medium or the sources. Blind source separation (BSS) has applications in several areas, such as communication, speech and audio processing, biomedical engineering, geophysical data processing, etc. [1]. BSS of instantaneous mixtures has attracted so far a lot of attention due to its many potential applications and its mathematical tractability that leads to several nice and simple BSS solutions [1–5].

Assume that m narrow band signals impinge on an array of $n \geqslant m$ sensors. The measured array output is a weighted superposition of the signals, corrupted by additive noise, i.e.

$$\boldsymbol{x}(t) = \boldsymbol{y}(t) + \boldsymbol{\eta}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{\eta}(t) \tag{1}$$

where $\boldsymbol{s}(t) = [s_1(t), \ldots, s_m(t)]^{\mathrm{T}}$ is the $m \times 1$ complex *source vector*, $\boldsymbol{\eta}(t) = [\eta_1(t), \ldots, \eta_n(t)]^{\mathrm{T}}$ is the $n \times 1$ complex *noise vector*, $\boldsymbol{A}$ is the $n \times m$ full column rank *mixing matrix*, and the superscript T denotes the transpose operator. The source signal vector $\boldsymbol{s}(t)$ is assumed to be a multivariate stationary complex stochastic process.

In this paper, we only consider the second-order BSS methods. Hence, the component processes $s_i(t)$, $1 \leqslant i \leqslant m$ are assumed to be temporally coherent and mutually uncorrelated, with zero mean and second-order moments:

$$\boldsymbol{S}(\tau) \stackrel{\text{def}}{=} E(\boldsymbol{s}(t+\tau)\boldsymbol{s}^{\star}(t)) = \operatorname{diag}[\rho_1(\tau), \ldots, \rho_m(\tau)] \tag{2}$$

where $\rho_i(\tau) \stackrel{\text{def}}{=} E(s_i(t+\tau)s_i^*(t))$, the expectation operator is E, and superscripts $*$ and $\star$ denote the conjugate of a complex and the complex conjugate transpose of a vector, respectively. The additive noise $\boldsymbol{\eta}(t)$ is modeled as a white stationary zero-mean complex random process of covariance $E(\boldsymbol{\eta}(t)\boldsymbol{\eta}^{\star}(t)) = \sigma^2\boldsymbol{Q}$. The latter matrix is proportional to identity (i.e. $E(\boldsymbol{\eta}(t)\boldsymbol{\eta}^{\star}(t)) = \sigma^2\boldsymbol{I}$) when the noise is spatially white. Under these assumptions, the observed data correlation matrices are given by

$$\boldsymbol{R}_x(\tau) = \boldsymbol{A}\boldsymbol{S}(\tau)\boldsymbol{A}^{\star} + \delta(\tau)\sigma^2\boldsymbol{Q}$$

From this expression, one can observe that the noise free correlation matrices are "diagonalizable" under the linear transformation $\boldsymbol{B} = \boldsymbol{A}^{\#}$ where the superscript $(\cdot)^{\#}$ is Moore–Penrose's pseudo-inversion operator, i.e. $\boldsymbol{B}\boldsymbol{R}_x(\tau)\boldsymbol{B}^{\mathrm{H}}$ are diagonal $\forall \tau \neq 0$. Hence, the source separation is achieved by decorrelating the signals at different time lags.

Before all, note that complete blind identification of separating (demixing) matrix $\boldsymbol{B}$ (or the equivalently mixing matrix $\boldsymbol{A}$) is impossible in the blind context, because the exchange of a fixed scalar between the source signal and the corresponding column of $\boldsymbol{A}$ leaves the observations unaffected. Also note that the signals *numbering* is immaterial. It follows that the best that can be done is to determine $\boldsymbol{B}$ up to a permutation and scalar shifts of its columns [3], i.e. $\boldsymbol{B}$ is a separating matrix if and only if

$$\boldsymbol{B}\boldsymbol{y}(t) = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{s}(t) \tag{3}$$

where $\boldsymbol{P}$ is a permutation matrix and $\boldsymbol{\Lambda}$ a non-singular diagonal matrix.

Under the above assumptions, we provide a general framework for the BSS including the study of the identifiability and its testing as well as the introduction of a simple but efficient decorrelation method and its performance analysis. The paper is organized as follows. Section 2 reviews the principal contributions to the BSS problem using the second-order statistics (SOS). Section 3 states the necessary and sufficient second-order identifiability conditions. We then propose an identifiability testing based on resampling technique in Section 4. Section 5 proposes a BSS algorithm using relative gradient technique. Section 6 is devoted to the performance analysis of the considered decorrelation method and the validation of the identifiability testing technique. Section 7 is for concluding remarks.