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Beamspace direction finding based on the conjugate gradient and the auxiliary vector filtering algorithms ☆, ☆☆

Jens Steinwandt [a,*], Rodrigo C. de Lamare [b], Martin Haardt [a]

[a] Communications Research Laboratory, Ilmenau University of Technology, P.O. Box 100565, 98684 Ilmenau, Germany
[b] Communications Research Group, Department of Electronics, University of York, York YO10 5DD, United Kingdom

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ABSTRACT

Motivated by the performance of the direction finding algorithms based on the auxiliary vector filtering (AVF) method and the conjugate gradient (CG) method as well as the advantages of operating in beamspace (BS), we develop two novel direction finding algorithms for uniform linear arrays (ULAs) in the beamspace domain, which we refer to as the BS AVF and the BS CG methods. The recently proposed Krylov subspace-based CG and AVF algorithms for the direction of arrival (DOA) estimation utilize a non-eigenvector basis to generate the signal subspace and yield a superior resolution performance for closely spaced sources under severe conditions. However, their computational complexity is similar to the eigenvector-based methods. In order to save computational resources, we perform a dimension reduction through the linear transformation into the beamspace domain, which additionally leads to significant improvements in terms of the resolution capability and the estimation accuracy. A comprehensive complexity analysis and simulation results demonstrate the excellent performance of the proposed algorithms and show their computational requirements. As examples, we investigate the efficacy of the developed methods for the discrete Fourier transform (DFT) and the discrete prolate spheroidal sequences (DPSS) beamspace designs.

1. Introduction

The need for the direction of arrival (DOA) estimation of incident signal wavefronts using sensor arrays is encountered in a broad range of important applications, including radar, wireless communications, biomedicine, etc. As a result, numerous methods for estimating the DOAs of signals have been proposed in the last few decades [1]. Among the most powerful techniques are the subspace-based algorithms, such as MUSIC [2], Root-MUSIC [3] and ESPRIT [4], which are proven to yield high-resolution capabilities. However, they require an eigendecomposition of the $M \times M$ spatial covariance matrix $\boldsymbol{R} = \mathbb{E}\{\boldsymbol{x}\boldsymbol{x}^H\}$ of the received data, corresponding to M sensor elements. As this is a computationally expensive operation with $\mathcal{O}(M^3 + M^2 N)$ multiplications, a new class of subspace-based DOA estimation methods termed Krylov subspace-based methods [5,6], adopting the auxiliary vector filtering (AVF) algorithm [7] or, as an extension, the conjugate gradient (CG) algorithm [8], was recently proposed. Note that another class of DOA

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* Corresponding author. Tel.: +49 3677 69 2613; fax: +49 3677 69 1195.
E-mail addresses: Jens.Steinwandt@tu-ilmenau.de (J. Steinwandt), rcdl500@ohm.york.ac.uk (R.C. de Lamare), martin.haardt@tu-ilmenau.de (M. Haardt).
URL: http://www.tu-ilmenau.de/crl (J. Steinwandt).

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estimators that do not resort to an eigendecomposition but still demand a high computational cost comprises the maximum-likelihood (ML) methods [1,9,10]. Here, however, we only focus on the subspace-based algorithms.

The advantage of the Krylov-based techniques is that they are applicable to arbitrary array geometries and avoid the eigendecomposition by iteratively generating an extended Krylov signal subspace that consists of the true signal subspace and the scanning vector itself. While the AVF algorithm forms the signal subspace from auxiliary vectors, the CG method applies residual vectors to span the Krylov subspace. Then, the unknown DOAs are determined by the search for the rank collapse of the extended signal subspace in the entire spatial spectrum, which occurs when the scanning vector is contained in it. This results in superior resolution performance for closely spaced sources under severe conditions, i.e., in the case of a low signal-to-noise ratio (SNR) and a small data record. However, despite utilizing a non-eigenvector basis they suffer from a similar computational complexity as the eigenvector-based methods, since the Krylov signal subspace is constructed for each search angle.

One way of significantly reducing the computational complexity is beamspace (BS) processing [11], which transforms the original data in element space into a reduced-dimensional subspace and performs the DOA estimation only in a spatial sector rather than in the entire angle range. Apart from the great computational savings, operation in beamspace also increases the resolution abilities as well as the estimation accuracy [12]. This is achieved by the enhancement of the SNR within the spatial sector of interest in analogy to beamforming. However, it is well known that beamspace processing does not improve the best achievable estimation accuracy as it only preserves the signals in the sector of interest, i.e., the corresponding Cramér–Rao lower bound (CRLB) in beamspace is equal to the element-space CRLB if there are only in-sector sources [13,14]. Beamspace techniques with robustness against strong sources that are located outside of the spatial sector were developed in [15,16].

In this paper, we propose two beamspace direction finding algorithms based on the CG and the AVF algorithms. Note that although these two Krylov-based algorithms are applicable to arbitrary array geometries we focus on uniform linear arrays (ULA) in this work to simplify the operation in beamspace. A generalization to other geometries can be achieved by considering array linearization techniques [17] prior to the proposed beamspace algorithms. Also, our methods are designed for the one-dimensional DOA estimation but extensions to the two-dimensional case are possible. For convenience, we assume that the number of signal sources d is known and that they are well inside the subband of interest. As the search for the signals is only conducted in a spatial sector, either a priori information of the approximate position of the DOAs is required or parallel processing of overlapping sectors of the angle spectrum has to be applied. We show that the proposed algorithms require a substantially lower computational complexity compared to their counterparts in element space. Moreover, they provide a better resolution and better estimation capabilities compared to previously developed beamspace algorithms, such as BS MUSIC [12], BS Root-MUSIC [18], and BS ESPRIT [19]. In addition, two different designs of the beamspace transformation matrix using the discrete Fourier transform (DFT) and discrete prolate spheroidal sequences (DPSS) are evaluated and compared.

The remainder of this paper is organized as follows. Section 2 describes the system model. The two different ways of designing the beamspace matrix are introduced and compared in Section 3. In Section 4, the proposed BS CG and BS AVF algorithms are presented, whereas Section 5 deals with the complexity analysis. Section 6 illustrates and discusses the simulation results and finally, the concluding remarks are drawn in Section 7.

Notation: We use lowercase boldface letters for column vectors and uppercase boldface letters for matrices. The superscripts T,* and H denote transpose, complex conjugate, and conjugate transpose, respectively, $\|\boldsymbol{x}\|$ represents the 2-norm of the vector $\boldsymbol{x}$ and $\mathbb{E}\{\cdot\}$ stands for the statistical expectation.

2. System model and beamspace processing

Let an M-element ULA receive narrowband signals originating from d $(d<M)$ far-field sources with the DOAs $\boldsymbol{\theta}=[\theta_1,\ldots,\theta_d]^T$. The ith of N available data snapshots of the $M\times 1$ array output vector can be modeled as

$$\boldsymbol{x}(i)=\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(i)+\boldsymbol{n}(i),\quad i=1,\ldots,N, \tag{1}$$

where $\boldsymbol{A}(\boldsymbol{\theta})=[\boldsymbol{a}(\theta_1),\ldots,\boldsymbol{a}(\theta_d)]\in\mathbb{C}^{M\times d}$ is the array steering matrix, $\boldsymbol{s}(i)=[s_1(i),\ldots,s_d(i)]^T\in\mathbb{C}^{d\times 1}$ represents the zero-mean vector of signal waveforms, and $\boldsymbol{n}(i)\in\mathbb{C}^{M\times 1}$ is the vector of white circularly symmetric complex Gaussian sensor noise with zero mean and variance σ_n^2. The $M\times 1$ steering vector $\boldsymbol{a}(\theta_l)$ corresponding to the lth source, $l=1,\ldots,d$, is expressed as

$$\boldsymbol{a}(\theta_l)=\left[1\quad e^{j2\pi(\Delta/\lambda_c)\sin\theta_l}\quad\ldots\quad e^{j2\pi(M-1)(\Delta/\lambda_c)\sin\theta_l}\right]^T, \tag{2}$$

where Δ denotes the interelement spacing of the ULA, λ_c is the signal wavelength, and omni-directional sensors have been assumed for the sake of notational simplicity. Using the fact that $\boldsymbol{s}(i)$ and $\boldsymbol{n}(i)$ are modeled as uncorrelated random variables, the $M\times M$ covariance matrix is calculated by

$$\boldsymbol{R}=\mathbb{E}\{\boldsymbol{x}(i)\boldsymbol{x}^H(i)\}=\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{R}_{ss}\boldsymbol{A}^H(\boldsymbol{\theta})+\sigma_n^2\boldsymbol{I}_M, \tag{3}$$

where $\boldsymbol{R}_{ss}=\mathbb{E}\{\boldsymbol{s}(i)\boldsymbol{s}^H(i)\}$ and $\boldsymbol{I}_M$ is the $M\times M$ identity matrix. In practice, the unknown covariance matrix is estimated by the sample covariance matrix

$$\hat{\boldsymbol{R}}=\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}(i)\boldsymbol{x}^H(i). \tag{4}$$

The linear transformation of the original data into the beamspace of a lower dimension B with $d<B<M$ is defined as

$$\tilde{\boldsymbol{x}}(i)=\boldsymbol{W}^H\boldsymbol{x}(i)\in\mathbb{C}^{B\times 1}, \tag{5}$$

where $\boldsymbol{W}$ is the $M\times B$ beamspace matrix satisfying $\boldsymbol{W}^H\boldsymbol{W}=\boldsymbol{I}_B$, so that the beamspace sensor noise remains spatially white. If the beamspace matrix $\boldsymbol{W}_o$ is not