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Multi-channel filter banks associated with linear canonical transform ☆

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ABSTRACT

The linear canonical transform (LCT) has been shown to be a powerful tool for optics and signal processing. This paper investigates multi-channel filter banks associated with the LCT. First, the perfect reconstruction (PR) conditions are analyzed and design method of PR filter banks for the LCT is proposed, which demonstrates that the LCT based filter banks can inherit conventional design methods of filter banks in the Fourier domain. Then polyphase decompositions in the LCT domain are defined and polyphase realization of the LCT based filter banks is derived in terms of polyphase matrices. Furthermore, multi-channel cyclic filter banks associated with the LCT are proposed by defining circular convolution in the LCT domain. The PR design method and polyphase representation of cyclic filter banks for the LCT are derived similarly. Finally, simulations validate the proposed design methods of the LCT based filter banks and also demonstrate potential application of the LCT based cyclic filter banks in image subband decomposition.

The linear canonical transform (LCT) has recently attracted much attention in the area of signal processing and optics [1–3], which is an integral transform with four parameters a,b,c,d and many transforms such as the Fourier transform (FT), the fractional Fourier transform (FrFT) and the Fresnel transform (FRT) are its special cases. The LCT of a signal $x(t)$ with parameter $M=(a,b,c,d)$ is defined as [2]

$$X_M(u)=L_{(a,b,c,d)}(x(t))(u)=\begin{cases}\sqrt{\frac{1}{j2\pi b}}e^{j\frac{d}{2b}u^2}\int_{-\infty}^{+\infty}x(t)e^{j\frac{a}{2b}t^2}e^{-j\frac{1}{b}ut}dt, & b\neq 0\\ \sqrt{d}e^{j\frac{cd}{2}u^2}x(du), & b=0\end{cases} \quad (1)$$

where $a,b,c,d \in R$ and $ad-bc=1$. When $(a,b,c,d)=(\cos\alpha, \sin\alpha, -\sin\alpha, \cos\alpha)$, the LCT reduces to the FrFT, i.e.,

$$L_{(\cos\alpha,\ \sin\alpha,\ -\sin\alpha,\ \cos\alpha)}(x(t))(u)=\sqrt{e^{-j\alpha}}X_\alpha(u) \quad (2)$$

where $X_\alpha(u)$ is the FrFT of the signal $x(t)$ with order α. For further details about the definition and properties of the LCT, [1–3] can be referred. Since the LCT has three free parameters, it is more flexible and has been found many applications in radar system analysis, filter design, phase retrieval, pattern recognition, encryption and many other areas [1–6].

As a generalization of the FT and FrFT, the basic theories of the LCT have been developed including uncertainty principles [7–11], convolution theorem [12,13], Hilbert transform [14,15], sampling theory [16–21], discretization [22–24] and so on, which can enrich the theoretical framework of the LCT and advance the application of the LCT. The theory of filter banks can be applied in almost any domain, which can separate a signal into a set of subband signals or combine many such subband signals into a single composite signal, and realize multi-resolution analysis of signals in the corresponding domain. Since the LCT has shown to be a powerful signal processing tool, it is necessary to study the filter banks theory associated with the LCT. Recently some researchers have studied the filter banks in the FrFT and LCT domains in order to realize subband decomposition of those signals that may be band limited in a certain FrFT or LCT

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domain. Meng et al. [25] has given the analysis of cyclic filter banks in the FrFT domain. Shinde has proposed two-channel paraunitary filter banks based on the LCT [26]. However multi-channel filter banks associated with the LCT has never been presented before.

In this paper we investigate L-channel filter banks associated with the LCT including the design method of perfect reconstruction (PR) filter banks and polyphase decomposition in the LCT domain. Furthermore, cyclic filter banks associated with the LCT are also proposed by defining circular convolution in the LCT domain, which are appropriate for dealing with finite length signals such as images. The simulations verify the effectiveness of the proposed design methods of filter banks in the LCT domain and also discuss potential application of the LCT based cyclic filter banks in image subband decomposition.

The rest of this paper is organized as follows. In Section 1 we consider L-channel filter banks associated with the LCT. The conditions for PR reconstruction are analyzed and the design method of filter banks for the LCT is proposed. Moreover, polyphase decompositions in the LCT domain are given. In Section 2 L-channel circular filter banks associated with the LCT are proposed. The PR design method and polyphase representation of cyclic filter banks for the LCT are also presented. In Section 3 some simulated examples are given to verify the achieved results. Finally we make conclusions in Section 4.

1. L-channel filter banks associated with the LCT

1.1. Preliminaries

Let $x(n)$ be a discrete time signal sampled from the continuous signal $x_c(t)$ with sampling interval Δt, i.e., $x(n)=x_c(n\Delta t)$. The discrete time LCT (DTLCT) of $x(n)$ with parameter $M=(a,b,c,d)$ is defined as [20]

$$\tilde{X}_M(\omega)=e^{j(d/2b)(b\omega/\Delta t)^2}\sum_{n=-\infty}^{n=+\infty}x(n)e^{j(a/2b)n^2\Delta t^2}e^{-jn\omega\,\mathrm{sgn}(b)} \tag{3}$$

where ω is the digital frequency in the LCT domain and sgn(b) is sign of b. Without loss of generality, we assume that $b>0$ in this paper.

Expander and decimator are two basic operations in multirate digital signal processing. Let $y(n)=x(n)\uparrow L$ denotes expander by an integer L, the DTLCT of $y(n)$ with parameter $M=(a,b,c,d)$ is [20]

$$\tilde{Y}_M(\omega)=\tilde{X}_M(L\omega) \tag{4}$$

Let $y(n)=x(n)\downarrow N$ denotes decimator by an integer N, the DTLCT of $y(n)$ with parameter $M=(a,b,c,d)$ is [20]

$$\tilde{Y}_M(\omega)=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}_M\left(\frac{\omega-2\pi k}{N}\right)e^{j2\pi kbd(\omega-\pi k)/(N\Delta t)^2} \tag{5}$$

The convolution is a fundamental operator in filter banks. The convolution of $x(n)$ and $h(n)$ for the LCT with parameter $M=(a,b,c,d)$ is defined as [12,26]

$$y(n)=x(n)\Theta h(n)=e^{-j(a/2b)n^2\Delta t^2}[x(n)e^{j(a/2b)n^2\Delta t^2}*h(n)e^{j(a/2b)n^2\Delta t^2}]=\sum_{k=-\infty}^{k=+\infty}h(k)x(n-k)e^{-j(a/b)\Delta t^2k(n-k)} \tag{6}$$

where $*$ denotes the conventional convolution. The DTLCT of convolution $y(n)$ is

$$\tilde{Y}_M(\omega)=\tilde{X}_M(\omega)\tilde{H}_M(\omega)e^{-j(db/2\Delta t^2)\omega^2} \tag{7}$$

The time delay operator is relevant to polyphase decomposition and PR of filter banks. The time delay operator for the LCT with parameter $M=(a,b,c,d)$ on a signal $x(n)$ is defined as [26]

$$D^k[x](n)=x(n-k)e^{j(a/2b)\Delta t^2(-2nk+k^2)} \tag{8}$$

Note that the delay operator is relevant with the sampling interval Δt. The delay operator $D[\cdot]$ has the following properties [26]:

$$D^l[D^k[x]](n)=D^{l+k}[x](n) \tag{9}$$

$$\text{If } y(n)=D^k[x](n),\quad \text{then}\quad \tilde{Y}_M(\omega)=e^{-jk\omega}\tilde{X}_M(\omega) \tag{10}$$

$$\text{If } y(n)=x(n)\Theta h(n),\quad \text{then}\quad D^l[x](n)\Theta D^k[h](n)=D^{l+k}[y](n) \tag{11}$$

1.2. Design of L-channel PR filter banks associated with the LCT

At first we show that the time delay operator given by Eq. (8) can be written as the convolution of $x(n)$ and $\delta(n-k)$ in the LCT domain.

Lemma 1. The time delay operator $D[\cdot]$ on a signal $x(n)$ can be written as

$$D^k[x](n)=(x(n)\Theta\delta(n-k))e^{-j(a/2b)k^2\Delta t^2} \tag{12}$$

Proof. Using Eq. (6), we have $x(n)\Theta\delta(n-k)=x(n-k)e^{-j(a/b)\Delta t^2k(n-k)}$. Thus, it is obvious that Eq. (12) holds.□

Lemma 1 means that the time delay operator $D[\cdot]$ is consistent with that defined by the convolution with δ function in [25]. We take the time delay operator $D[\cdot]$ in this paper because it has the property given by Eq. (9).

The structure of L-channel filter banks associated with the LCT is shown in Fig. 1 with convolution as defined in Eq. (7), where $\tilde{H}_{l,M}(\omega)$ and $\tilde{G}_{l,M}(\omega)$ are the DTLCT of $h_{l,M}(n)$ and $g_{l,M}(n)$, respectively, $l=0,1,\ldots,L-1$. The subfilters $\{h_{l,M}(n), l=0, 1,\ldots,L-1\}$ are known as the analysis filters which split the original signal $x(n)$ into a number of subband signals in the LCT domain, and the subfilters $\{g_{l,M}(n), l=0,1,\ldots,L-1\}$ are known as the synthesis filter banks which combine the subband signals to reconstruct the original signal. The reconstructed signal $y(n)$ should be as close to the original input signal as possible in a certain well defined sense, i.e., $\tilde{Y}_M(\omega)=ce^{-jn_0\omega}\tilde{X}_M(\omega)$ or $y(n)=cD^{n_0}[x](n)$, where c,n_0 are constants and the corresponding LCT based filter banks are PR.

First we analyze the input–output relation of the filter banks in the LCT domain as shown in Fig. 1. Using Eqs. (4), (5) and (7), we obtain the DTLCT of the output of the filter