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Accurate estimation of common sinusoidal parameters in multiple channels

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ABSTRACT

Parameter estimation for exponentially damped complex sinusoids in the presence of white noise using multiple channel measurements is addressed. More precisely, we are interested in the damping factor and frequency parameters which are common among all channels. By exploiting linear prediction and weighted least squares technique, an iterative algorithm is devised to extract the common dynamics of the cisoids. Statistical analysis of the proposed method is studied and confirmed by computer simulations. Moreover, it is shown that the developed estimator attains optimum estimation accuracy and is superior to a conventional subspace-based algorithm when the signal-to-noise ratio is sufficiently high.

1. Introduction

Parameter estimation for sinusoidal signals embedded in additive noise is an important research topic because of its wide applications in science and engineering [1–4]. The parameters of interest are the damping factors, frequencies, amplitudes and phases. Apart from conventional estimation from only one data set [5–9], the problem of finding the sinusoidal parameters from multiple channels has also received considerable attention [10–12]. A representative application is to extract the common epileptiform activity [10,11] from the artifacts and background activity using multiple electroencephalography (EEG) recordings. While in nuclear magnetic resonance (NMR) spectroscopy [12], quantification of the complex time-domain signals is useful for brain tumor detection and material health monitoring.

In this paper, we address multi-channel sinusoidal parameter estimation of [10,11] where each channel output is modeled as a sum of exponentially damped complex sinusoids in white noise. Among all channels, there are some cisoids whose damping factors and frequencies are common, which correspond to the common dynamics. Note that the corresponding amplitudes and phases can be easily determined channel by channel according to a linear least squares fit once the damping factor and frequency parameters are estimated. This problem is more challenging than that of [12] where all poles are common among the channels. With the use of shift-invariance of the signal subspace and total least squares technique, a subspace-based algorithm for estimating the common dynamics using multiple channels is developed in [13]. In [14], a solution based on the orthogonality of the signal and noise subspaces, which belongs to the family of the orthogonal vector method, is devised. Though computationally attractive, these algorithms cannot give optimum estimation performance. In this work, our main contribution is to devise an estimator for the common sinusoidal parameters with variance attaining the Cramér–Rao lower bound (CRLB) under sufficiently small noise conditions.

The rest of the paper is organized as follows. The problem of estimating the common damping factors and frequencies from multiple channel measurements is formulated in Section 2. Based on linear prediction (LP)

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and weighted least squares (WLS) technique, the algorithm is devised in Section 3. The mean and variance of the proposed estimator are derived in Section 4. Simulation results are included in Section 5 to corroborate the analytical development and to evaluate the performance of the proposed approach by comparing with [13] as well as CRLB. Finally, conclusions are drawn in Section 6.

1.1. Symbols and notations

Throughout this paper, we use boldfaced uppercase letters to denote matrices, boldfaced lowercase letters for column vectors, and lowercase letters for scalar quantities. Superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ and $(\cdot)^\dagger$ represent complex conjugate, transpose, Hermitian transpose, matrix inverse, and pseudo-inverse, respectively. The notation $\mathbf{A} \in \mathbb{C}^{M\times N}$ means that $\mathbf{A}$ is a complex $M\times N$ matrix. Moreover, $\mathbb{R}(a)$ and $\angle(a)$ denote the real part and phase angle of a. The block diagonal matrix, with $\mathbf{A}_1$ and $\mathbf{A}_2$ being its components, is denoted by diag $(\mathbf{A}_1,\mathbf{A}_2)$, and vec$(\mathbf{A})$ is the columnwise vectorized version of matrix $\mathbf{A}$. The Kronecker product is denoted by $\otimes$, $\mathbf{I}_M$ is the identity matrix of dimension M and $\mathbf{0}_{M\times N}$ is the $M\times N$ zero matrix. Moreover, $\boldsymbol{\xi}_{M,i}$ denotes the ith column of $\mathbf{I}_M$. The $\mathbf{H}_{M,u}$ and $\mathbf{H}_{M,\ell}$ denote matrices containing the first and last $(M-1)$ rows of $\mathbf{I}_M$, respectively. The Hankel matrix with first column $\mathbf{a}$ and last row $\mathbf{b}^T$ is denoted by Hankel $(\mathbf{a},\mathbf{b}^T)$. Similarly, Toep $(\mathbf{a},\mathbf{b}^T)$ represents the corresponding Toeplitz matrix. In addition, $\delta_{i,j}$ is the Dirac delta function and $\mathcal{CN}(\mathbf{x};\boldsymbol{\mu},\boldsymbol{\Sigma})$ is the complex Gaussian probability density function with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. Furthermore, var (a) and cov $(\mathbf{a})$ represent the variance of a and covariance of $\mathbf{a}$, respectively. In addition, $\mathbb{E}$ isthe expectation operator. In the following, we estimate $\mathbf{a}$ by minimizing a cost function. In this situation, $\mathbf{a}$ is a variable instead of a fixed-value constant. Therefore, we denote $\breve{\mathbf{a}}$ by the variable to avoid confusion. Finally, $\hat{\mathbf{a}}$ represents the estimate of $\mathbf{a}$.

2. Problem formulation

The observed signal of the kth channel at time n is modeled as

$$x_{k,n} = s_{k,n} + q_{k,n}, \quad k=1,\ldots,K, \ n=1,\ldots,N, \tag{1}$$

where

$$s_{k,n} = \sum_{m=1}^{M_k} \alpha_{k,m}\beta_{k,m}^n, \quad \beta_{k,m} = \rho_{k,m}\exp\{j\omega_{k,m}\}. \tag{2}$$

Here, K, N and M_k are the number of channels, data length and number of damped exponentials in the kth channel, respectively. The damping factor, frequency and complex amplitude of the mth sinusoid in the kth channel are denoted by $\rho_{k,m}<1$, $\omega_{k,m}\in(0,2\pi)$ and $\alpha_{k,m}$, respectively. Without loss of generality, let $\omega_{1,m}=\cdots=\omega_{K,m}=\omega_m$ and $\rho_{1,m}=\cdots=\rho_{K,m}=\rho_m$, $m=1,\ldots,M$ be the common dynamics while $M_1\le\cdots\le M_K$ and $M<M_1$. It is assumed that M and $\{M_k\}$ are known *a priori*. Furthermore, the noise $q_{k,n}$ is independent and identically distributed complex Gaussian random process with mean zero and variance σ^2. Note that if the signals arrive at different times in different channels, we first need to determine, in each channel, the time index after which the signal is present, referred to as signal start time. Technically, a hypothesis test is carried out to detect the incoming data whether it consists of signal plus noise or noise only with certain confidence level. We then only utilize the data after the signal start time and discard the previous data. In this paper, we assume that the detection has been performed and focus on estimating the common dynamics, that is, ω_m and ρ_m, $m=1,\ldots,M$ from $x_{k,n}$, $k=1,\ldots,K$, $n=1,\ldots,N$.

3. Algorithm development

In this section, we develop a three-step iterative algorithm to estimate the common dynamics through solving LP coefficients of all channels using weighted least squares techniques. It is well known that the complex sinusoid $s_{k,m}$ can be uniquely expressed as a linear combination of its previous M_k samples [15]:

$$\sum_{m=0}^{M_k} a_{k,m}s_{k,n-m} = 0, \quad a_{k,0}=1, \ k=1,\ldots,K, \tag{3}$$

where $a_{k,m}$, $m=1,\ldots,M_k$ are the LP coefficients for the kth channel, which are directly related to $\beta_{k,m}$, $m=1,\ldots,M_k$. That is, $z=\beta_{k,m}$ satisfies the following polynomial:

$$\sum_{m=0}^{M_k} a_{k,m}z^{M_k-m} = 0, \quad k=1,\ldots,K. \tag{4}$$

It indicates that once $\mathbf{a}_{k,m}$ is obtained, $\beta_{k,m}$ can be estimated by solving the polynomial equation of (4). In particular, the common dynamics, β_m, $m=1,\ldots,M$, are the common roots of the K equations in (4). Writing this relationship in matrix form yields

$$\mathbf{D}^T\mathbf{C} = \mathbf{0}_{M\times K} \tag{5}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{(M_K-M_1)\times 1} & \mathbf{0}_{(M_K-M_2)\times 1} & \cdots & 1 \\ 1 & 1 & \ddots & \\ \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_K \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} \beta_1^{M_K} & \beta_2^{M_K} & \cdots & \beta_M^{M_K} \\ \beta_1^{M_K-1} & \beta_2^{M_K-1} & \cdots & \beta_M^{M_K-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix},$$

$$\mathbf{a}_k = [a_{k,1} \ \cdots \ a_{k,M_k}]^T.$$

Here, we append zeros in $\mathbf{C}$ because the numbers of tones in each channel are generally different. It is noticed that $\{\beta_m\}$ can be obtained from $\mathbf{D}$ via the relationship of

$$\mathbf{D}_\ell\mathbf{P} = \mathbf{D}_u, \tag{6}$$

where $\mathbf{D}_\ell$ and $\mathbf{D}_u$ are matrices containing the lower and upper M_K rows of $\mathbf{D}$, respectively, and $\mathbf{P}=\text{diag}(\beta_1,\ldots,\beta_M)$. In practice, we substitute the observed $x_{k,n}$ for the unavailable $s_{k,n}$ into (3) and replace the equal sign by the approximately equal sign. Denote the left-hand side of (3) by LP error. Then, the overall LP error for all the K