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Sequency-ordered generalized Walsh–Fourier transform

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A new transform family, called the sequency-ordered generalized Walsh–Fourier transform (SGWFT), is proposed in this paper. Using the kernel matrix generation process and the controllable phase quantization parameter, the Walsh–Hadamard transform (WHT), the sequency-ordered Hadamard transform (SCHT), and the discrete Fourier transform (DFT) become special cases of the SGWFT. The SGWFT can be adjusted by a single parameter to become the WHT, the SCHT, and the DFT. In addition, the SGWFT also has the radix-2 and the split-radix fast algorithms. Compared with the WHT and the SCHT, the properties and the performance of the SGWFT are more similar to those of the DFT. On the other hand, compared with the DFT, the number of multiplications in the SGWFT is less. We also show that the proposed SGWFT has better performance in the applications of DS-CDMA sequence design and transform coding.

1. Introduction

The discrete orthogonal transform (DOT) is widely used in signal analysis, communication, and image processing [1–7,17,18,28]. Two of the most important fundamental DOTs are the Walsh–Hadamard transform (WHT) and the discrete Fourier transform (DFT). They are used in many applications. For example, the WHT is the kernel element for establishing the code division multiple access (CDMA) system and the DFT is the key component of the OFDM system. Moreover, both the WHT and the DFT have fast algorithms, such as the well-known radix-2 algorithm and the split-radix FFT algorithm, which are suitable for hardware implementation.

The WHT matrix contains only two values, 1 and -1. Therefore, it is suitable for multiplication- free implementation. In [8,9], the unified complex Hadamard transform (UCHT) was proposed. Instead of ± 1, the elements of the UCHT matrix can be 1, -1, i, and $-i$. Similar to the original WHT, the UCHT is efficient for implementation and has the fast algorithm with complexity $N\log_2 N$. However, the performance of the UCHT for signal analysis is superior to that of the WHT. Furthermore, the UCHT can reduce the cross correlation between the channels in CDMA.

Although there are many useful applications of the UCHT, the lack of a physical meaning of "frequency" makes it difficult to apply the UCHT in spectrum analysis. Therefore, the sequency-ordered complex Hadamard transform (SCHT) [10,11] was proposed. This transform uses the complex Radamacher function (CRAD), which contains only four values: 1, -1, i, $-i$, as the generating basis. The SCHT is constructed through periodic sampling and element-by-element vector multiplication of the CRAD. Unlike the case of the UCHT, the numbers of zero crossings in the complex plane of the SCHT vectors are arranged in ascending order. Therefore, the SCHT is suitable for spectrum analysis, because we can specify the "low" and "high" frequency components of the signal. Additionally, as compared to the spectrum of the WHT, the spectrum of the SCHT is closer to that of the DFT. In the application of DS-CDMA spreading sequence design [11], the bit-error rate (BER) of the SCHT sequence was

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found to be less than that of the UCHT, the WHT, or the Gold sequence in the asynchronous multipath fading channel case.

In [12], the conjugate symmetric sequency ordered Hadamard transform (CS-SCHT) was proposed. As the case of the SCHT, the rows of the CS-SCHT matrix are also arranged in ascending order. The difference between the SCHT and the CS-SCHT is that the CS-SCHT spectrum has the conjugate symmetry property for real input signals.

Moreover, in [27,30,32], the DFT and the WHT were combined and used in the application of orthogonal frequency division multiplexed (OFDM).

These generalized Walsh–Hadamard transforms have fast algorithms and can be easily implemented [8–12,31]. However, when regarding the DFT as the benchmark for signal analysis applications, none of the above transforms is a good approximation of the DFT.

Therefore, it is an important issue to find whether there is another operation with computational complexity less than that of the DFT and signal processing performance better or comparable to that of the DFT.

In this paper, a new family of DOTs, which is called the phase-quantization-based generalized Walsh–Fourier transform (SGWFT), is proposed. The SGWFT links the WHT, the SCHT, and the DFT all together using a single parameter. Unlike the UCHT, the SCHT, and the CS-SCHT, which have only four values on the unit circle, the SGWFT is controlled by a single parameter p, which quantizes the SGWFT kernel elements into p kinds of values. Therefore, the WHT ($p=2$), the SCHT ($p=4$) and the DFT ($p=N$) can all be treated as the special cases of the SGWFT.

To generate the SGWFT matrix, we modify the generation process in [10] and adopt a new generalized generating function $W(t)$ for creating the DOT matrix. The generalized Rademacher function (GRF) proposed in [11] can be viewed as a special group of $W(t)$. Then, the SGWFT matrix is created by the GRF and our proposed DOT generation process. Because the behavior of the SGWFT lies among those of the WHT, the SCHT and the DFT, it can balance the trade-offs in these transforms and still have the orthogonality property, the sequency-ordering property, and the radix-2 fast algorithm. Meanwhile, the behavior of the SGWFT can easily be switched among those of the WHT, the SCHT, and the DFT simply by adjusting the control parameter for real time applications.

Compared with the WHT and the SCHT, the SGWFT magnitude spectrum resist shift change more robustly and can approximate the DFT spectrum more closely. Compared with the CS-SCHT, although the SGWFT lacks the conjugate symmetry property in general, it has the reciprocal inverse property, as does the reciprocal-orthogonal parametric transform [13–16]. This property makes it possible to use the same implementation structure to realize either the forward or the inverse SGWFTs. On the other hand, compared with the DFT, because of the phase quantization property, the SGWFT requires fewer twiddle factors and can save the number of multiplications and memory requirement. Moreover, we will show that, in the applications of CDMA sequence design and transform coding, the SGWFT can outperform the WHT, the SCHT, and the DFT.

This paper is organized as follows. In Section 2, we briefly review the SCHT in [10]. In Section 3, we generalize the method in [10] and propose a new DOT matrix generation process. In Section 4, we propose the SGWFT, which is based on the generation process proposed in Section 3. Furthermore, in this section, we discuss the properties of the SGWFT. In Section 5, we apply the SGWFT to signal multiplexing and transform coding. We will show that the proposed SGWFT has superior performance in CDMA sequence design and has lower values of cross correlation (i.e., the mutual interference among different users) compared to the existing transforms. In Section 6 we make conclusions.

2. Review on sequency-ordered complex Hadamard transform

In [10], the authors use the complex Rademacher functions (CRAD) [26] to generate the sequency-ordered complex Hadamard transform (SCHT) matrices. We briefly review the generating procedure in this section. Over the time interval $0 \le t < 1$, the CRAD is defined as follows:

$$CRAD(0,t) = \begin{cases} 1, & t \in [0,1/4), \\ j, & t \in [1/4,1/2), \\ -1, & t \in [1/2,3/4), \\ -j, & t \in [3/4,1), \end{cases} \quad (1)$$

$$CRAD(0,t+1) = CRAD(0,t), \quad (2)$$

$$CRAD(r,t) = CRAD(0,2^r t), \quad (3)$$

where r is a non-negative integer.

When the length N is 2^n, we take samples on $CRAD(r, t)$ as follows:

$$R_n(r,k) = CRAD(r,(k/N+1/4N)) \text{ for } k = 0,1,\ldots,N-1. \quad (4)$$

From the above definitions, the SCHT matrix $\boldsymbol{H}$ is generated from the following equation:

$$H_N(m,k) = \prod_{r=0}^{n-1} R_n(r,k)^{m_r}. \quad (5)$$

The detail about the derivation of $\boldsymbol{H}$ can be seen from Eq. (7) in [10]. In (5), m_r is the binary representation coefficient of the row index m of $\boldsymbol{H}$, i.e.,

$$m = \sum_{r=0}^{l-1} m_r 2^r. \quad (6)$$

Since m_r is a value in the binary numeral system, its value is 0 or 1.

The matrix $\boldsymbol{H}$ was proven to have the orthogonality, the sequency-ordering, the unitary element value, the transpose symmetry and the radix-2 fast algorithm properties in [10].

3. General ways to construct discrete orthogonal transforms

We extend the previous work by using new generating functions to replace the CRAD in (1)–(3) to construct the new discrete orthogonal transform (DOT) matrix. We will