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Biquaternion cumulant-MUSIC for DOA estimation of noncircular signals ☆

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ABSTRACT

Direction-of-arrival (DOA) estimation for noncircular sources is addressed within the hypercomplex framework utilizing fourth-order (FO) cumulants and a MUSIC-like estimator is proposed. Simulation results show the better performance of the proposed method compared to its complex counterpart in terms of both accuracy and robustness to model errors due to the stronger orthogonality in the biquaternion domain.

1. Introduction

Direction-of-arrival (DOA) estimation for noncircular signals, which are omnipresent in radar and communication systems such as binary phase shift keying (BPSK) and amplitude modulation (AM) signals, has been raising increasing attention. Various algorithms based on second-order statistics, as well as high-order statistics (HOS), have been presented for DOA estimation. In [1], a MUSIC-like algorithm was proposed for noncircular signals (named NC-MUSIC) with increased processing capacity and then the spectrum searching was replaced for polynomial rooting in [2]. A detailed study of MUSIC-like algorithms for noncircular signals can be found in [3]. For centro-symmetric arrays, the unitary ESPRIT algorithm was meliorated for noncircular signals in [4]. An extended $2q$-MUSIC algorithm [5] for noncircular signals ($q=1$) based on second-order statistics and non-Gaussian noncircular signals ($q>1$) based on HOS, named NC-$2q$-MUSIC, was proposed in [6] and was shown to possess a further extended aperture and better performance compared with original algorithm.

In the array signal processing literature, signals are generally processed in complex numbers. The extended data structure acquired either physically (e.g., recorded by vector-sensors) or from signal processing (e.g., the case of receiving noncircular signals) is generally set out to be a prolonged vector. Recently, several algorithms for DOA estimation using vector-sensors were proposed in the hypercomplex framework using quaternions, biquaternions and quad-quaternions [7–10]. Instead of laying the data recorded by different components of a vector-sensor array on sequential places to create a "long vector", they relate them to different imaginary units to generate a hypercomplex vector. Due to the stronger orthogonality in the hypercomplex domain compared to the complex one, better performance is shown in both accuracy and robustness. Motivated by this fact, we herein propose a MUSIC-like algorithm in the hypercomplex framework using fourth-order (FO) cumulants. Though there are $2^4=16$ FO cumulant matrices, three of them are sufficient to achieve further aperture extension. To utilize the three matrices symmetrically, biquaternions are used.

The rest of the paper is organized as follows. In Section 2, we introduce the mathematical preliminaries on biquaternion algebra. We formulate the problem in Section 3 and propose our algorithm in Section 4. Some numerical examples to illustrate the performances of the proposed

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algorithm are given in Section 5 and we conclude the paper in Section 6.

2. Mathematical preliminaries

W.R. Hamilton's biquaternions [11,12] form an eight-dimensional space and can be viewed as "complexified quaternions" or "quartered complex numbers", namely, a biquaternion $b \in \mathbb{H}_{\mathbb{C}}$ possesses eight parts (one real and seven imaginary parts) and can be expressed by

$$\begin{aligned} b &= q_0 + \mathtt{I} q_1 \\ &= c_0 + \mathtt{i} c_1 + \mathtt{j} c_2 + \mathtt{k} c_3 \\ &= b_{00} + \mathtt{i} b_{10} + \mathtt{j} b_{20} + \mathtt{k} b_{30} + \mathtt{I} b_{01} + \mathtt{i}\mathtt{I} b_{11} + \mathtt{j}\mathtt{I} b_{21} + \mathtt{k}\mathtt{I} b_{31} \end{aligned} \tag{1}$$

where $\{q_n\}_{n=0}^{1}$, $\{c_n\}_{n=0}^{3}$, and $\{b_{nm}\}_{n=0,m=0}^{n=3,m=1}$ are quaternions, complex numbers with imaginary unit of $\mathtt{I}$, and real numbers, respectively. In this paper, we denote the sets of biquaternions, quaternions, complex numbers with imaginary unit of $\dot{u}$, and real numbers as $\mathbb{H}_{\mathbb{C}}$, $\mathbb{H}$, $\mathbb{C}_{\dot{u}}$, and $\mathbb{R}$, respectively, where $\dot{u} = \mathtt{i},\mathtt{j},\mathtt{k},\mathtt{I}$, which are subject to the following constraints:

$$\begin{aligned} &\mathtt{i}^2 = \mathtt{j}^2 = \mathtt{k}^2 = \mathtt{I}^2 = -1 \\ &\mathtt{ij} = -\mathtt{ji} = \mathtt{k}, \mathtt{jk} = -\mathtt{kj} = \mathtt{i}, \mathtt{ki} = -\mathtt{ik} = \mathtt{j} \\ &\mathtt{iI} = \mathtt{Ii}, \mathtt{jI} = \mathtt{Ij}, \mathtt{kI} = \mathtt{Ik} \end{aligned} \tag{2}$$

In addition, we use superscripts T, H, and $*$ to denote transpose, Hermitian transpose, and conjugation of a complex matrix. We then present a brief list of properties of biquaternions involved in formulating the algorithm.

(1) A biquaternion matrix $\boldsymbol{B} \in \mathbb{H}_{\mathbb{C}}^{M\times N}$ is defined as an $M \times N$ matrix with biquaternion entries, i.e.,

$$\begin{aligned} \boldsymbol{B} = \boldsymbol{B}_{00} + \mathtt{i}\boldsymbol{B}_{10} + \mathtt{j}\boldsymbol{B}_{20} + \mathtt{k}\boldsymbol{B}_{30} + \mathtt{I}\boldsymbol{B}_{01} \\ + \mathtt{iI}\boldsymbol{B}_{11} + \mathtt{jI}\boldsymbol{B}_{21} + \mathtt{kI}\boldsymbol{B}_{31} \end{aligned} \tag{3}$$

where $\{\boldsymbol{B}_{nm}\}_{n=0,m=0}^{n=3,m=1} \in \mathbb{R}^{M\times N}$.

(2) The total conjugate of biquaternion b and Hermitian transpose of biquaternion matrix $\boldsymbol{B}$ are respectively given by

$$\begin{aligned} b^{\diamond} = b_{00} - \mathtt{i} b_{10} - \mathtt{j} b_{20} - \mathtt{k} b_{30} - \mathtt{I} b_{01} \\ + \mathtt{iI} b_{11} + \mathtt{jI} b_{21} + \mathtt{kI} b_{31} \end{aligned} \tag{4}$$

$$\begin{aligned} \boldsymbol{B}^{\dagger} = \boldsymbol{B}_{00}^{\mathrm{T}} - \mathtt{i}\boldsymbol{B}_{10}^{\mathrm{T}} - \mathtt{j}\boldsymbol{B}_{20}^{\mathrm{T}} - \mathtt{k}\boldsymbol{B}_{30}^{\mathrm{T}} - \mathtt{I}\boldsymbol{B}_{01}^{\mathrm{T}} + \mathtt{iI}\boldsymbol{B}_{11}^{\mathrm{T}} \\ + \mathtt{jI}\boldsymbol{B}_{21}^{\mathrm{T}} + \mathtt{kI}\boldsymbol{B}_{31}^{\mathrm{T}} \end{aligned} \tag{5}$$

For two biquaternions $a,b \in \mathbb{H}_{\mathbb{C}}$ and two biquaternion matrices $\boldsymbol{A} \in \mathbb{H}_{\mathbb{C}}^{M\times N}, \boldsymbol{B} \in \mathbb{H}_{\mathbb{C}}^{N\times K}$, there holds [8]

$$(ab)^{\diamond} = b^{\diamond} a^{\diamond} \tag{6}$$

$$(\boldsymbol{AB})^{\dagger} = \boldsymbol{B}^{\dagger}\boldsymbol{A}^{\dagger} \tag{7}$$

(3) A square biquaternion matrix $\boldsymbol{B} \in \mathbb{H}_{\mathbb{C}}^{N\times N}$ is Hermitian if [12]

$$\boldsymbol{B} = \boldsymbol{B}^{\dagger} \tag{8}$$

(4) The norm of a biquaternion vector $\boldsymbol{b} \in \mathbb{H}_{\mathbb{C}}^{N\times 1}$ is defined as

$$\|\boldsymbol{b}\| = \sqrt{\mathcal{R}(\boldsymbol{b}^{\dagger}\boldsymbol{b})} \tag{9}$$

where operator $\mathcal{R}(\cdot)$ denotes the real part of a biquaternion and operator $\sqrt{\cdot}$ denotes the square-root operation [8].

(5) Two biquaternion vectors $\boldsymbol{a},\boldsymbol{b} \in \mathbb{H}_{\mathbb{C}}^{N\times 1}$ are orthogonal if [8]

$$\boldsymbol{a}^{\dagger}\boldsymbol{b} = 0 \tag{10}$$

(6) The eigenvalue decomposition (EVD) of a Hermitian biquaternion matrix $\boldsymbol{B} \in \mathbb{H}_{\mathbb{C}}^{N\times N}$ is given by

$$\boldsymbol{B} = \sum_{n=1}^{2N} \lambda_n \boldsymbol{u}_n \boldsymbol{u}_n^{\dagger} \tag{11}$$

where $\{\lambda_n\}_{n=1}^{2N} \in \mathbb{R}$ are real eigenvalues and $\{\boldsymbol{u}_n\}_{n=1}^{2N} \in \mathbb{H}_{\mathbb{C}}^{N\times 1}$ are unit-norm orthogonal eigenvectors [8].

(7) For a complex matrix $\boldsymbol{C} = \boldsymbol{C}_0 + \mathtt{i}\boldsymbol{C}_1$, where $\boldsymbol{C} \in \mathbb{C}_{\mathtt{i}}^{M\times N}$, $\boldsymbol{C}_0,\boldsymbol{C}_1 \in \mathbb{R}^{M\times N}$, there holds

$$\mathtt{jI}\boldsymbol{C} = \mathtt{jI}(\boldsymbol{C}_0 + \mathtt{i}\boldsymbol{C}_1) = (\boldsymbol{C}_0 - \mathtt{i}\boldsymbol{C}_1)\mathtt{jI} = \boldsymbol{C}^{*}\mathtt{jI} \tag{12}$$

3. Problem formulation

3.1. On second-order noncircularity

Second-order (SO) noncircular signals possess nonzero conjugate SO moments in addition to the SO moments. A particular kind of SO noncircular signals, called rectilinear signals, e.g., binary phase shift keying (BPSK) and amplitude modulation (AM) signals, are widely studied in the array signal processing literature. The SO moment and conjugate SO moment of a rectilinear signal $s(t) \in \mathbb{C}_{\mathtt{i}}$ are given by

$$E\{|s(t)|^2\} = \sigma^2 \tag{13}$$

$$E\{s^2(t)\} = \sigma^2 e^{\mathtt{i}\varpi} \tag{14}$$

respectively, where $\varpi \in [0,2\pi)$ is referred to as the noncircular phase and $\sigma^2 > 0$ denotes the power of $s(t)$.

We assume that $s(t)$ is also non-Gaussian, and it possesses a nonzero kurtosis which is defined as

$$\kappa = \mathrm{cum}\{s(t),s^{*}(t),s(t),s^{*}(t)\} = -2\sigma^4 \tag{15}$$

where operator "cum" denotes the FO cumulant and is defined as

$$\begin{aligned} &\mathrm{cum}\{s_1(t),s_2(t),s_3(t),s_4(t)\} \\ &\quad = E\{s_1(t)s_2(t)s_3(t)s_4(t)\} - E\{s_1(t)s_3(t)\}E\{s_2(t)s_4(t)\} \\ &\qquad - E\{s_1(t)s_4(t)\}E\{s_2(t)s_3(t)\} - E\{s_1(t)s_2(t)\}E\{s_3(t)s_4(t)\} \end{aligned} \tag{16}$$

For simplicity, we denote $\mathrm{cum}\{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{p}\}$ (where $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{p} \in \mathbb{C}_{\mathtt{i}}^{N\times 1}$) as a $N^2 \times N^2$ matrix of which the $((k_1-1)N+k_2, (k_3-1)N+k_4)$-th entry is $\mathrm{cum}\{x_{k_1},y_{k_2},z_{k_3},p_{k_4}\}$, where $\{k_n\}_{n=1}^{4} = 1,2,\ldots,N$.

3.2. Array model for noncircular sources

We assume an N-element scalar-sensor array illuminated by M statistically independent noncircular sources $\{s_m(t)\}_{m=1}^{M} \in \mathbb{C}_{\mathtt{i}}$ from $\{\theta_m\}_{m=1}^{M} \in [-\pi/2,\pi/2)$, steered by