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Review article

A critical analysis of design, facts, bias and inference in the approximate number system training literature: A systematic review

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ABSTRACT

A popular suggestion states that an evolutionarily grounded analogue magnitude representation, also called an approximate number system (ANS) or 'number sense' underlies human mathematical knowledge. During recent years many studies aimed to train the ANS with the intention of transferring improvements to symbolic arithmetic. Here we critically evaluate all published studies. We conclude that there is no conclusive evidence that specific ANS training improves symbolic arithmetic. We provide a citation analysis demonstrating that highly controversial results often get cited in support of specific claims without discussion of controversies. We suggest ways to run future training studies so that clear evidence can be collected and also suggest that data should be discussed considering both supporting and contrary evidence and arguments.

1. Introduction

A popular suggestion is that an evolutionarily grounded analogue magnitude representation, also called an approximate number system (ANS) or 'number sense', underlies human mathematical knowledge [8]. During recent years many studies aimed to train the ANS with the intention of transferring improvements to symbolic arithmetic. It is important to critically evaluate these studies because experience shows that interpretations are quickly taken up by researchers, practitioners and parents alike perhaps without much evaluation of how methods, results and study conclusions relate to each other, whereas usually the devil hides in the details. Unfortunately, many review papers tend to gloss over critical study details even though experimental design, analysis and/or inferential logic problems may inhibit clear conclusions or even disqualify results. Hence, in order to see clearly, here we critically review ANS training studies. We highlight both study-specific and general problems. We conclude that there is no conclusive evidence that specific ANS training improves symbolic arithmetic. We suggest ways to run future training studies so that clear evidence can be collected. We draw attention to the fact that highly controversial results often get cited in support of very specific claims in the literature without discussion of controversies. We suggest that this practice may facilitate the creation of a 'highly cited null field' which nevertheless gives an impression of positive results with regard to the ANS training literature. Below we first define important terms, then review studies one by one (because it is crucial to understand the details of individual studies so that they can be properly evaluated) and then draw some general conclusions. We especially point to the importance of bias-free discussion of results and placing them in the context of contrary as well as supportive literature.

1.1. What is number sense and the ANS?

A prerequisite of meaningful scientific debate is that we have a clear definition of what we wish to discuss. Literature regarding the ANS and number sense is often not up to this expectation as many researchers use this term in many different ways, and relevant definitions even seem to shift over time. Such confusions may result in some papers citing other papers as supporting evidence whereas they may have used completely different and non-compatible theoretical and/or operational definitions of number sense.

Here we assume that all the following terms mean the same: 'approximate number system', 'ANS', 'number sense', 'quantity representation', '(approximate) magnitude representation', '(approximate) analogue magnitude representation'. We take that all the above terms in the papers discussed below refer to the ANS in the sense defined by [8]. This concept can be defined as an ancient, evolutionarily grounded pre-human sense of magnitude which represents numerosity (the number of items) in a modality-independent and approximate manner and it enables magnitude discriminations. Consequently, it is often claimed that this ANS is the intuitive pre-cursor of all human mathematics [9]. It is to note that previously this concept was mostly called 'number sense', but more recently the tendency is to call it 'ANS'. It is also worth noting that the above ANS definition is very different

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Table 1
ANS definitions from the papers discussed. The studies are cited in the order of discussing them in this paper.

Citation	Definition
[74]; p2	*the ability to represent and manipulate numerical quantities non-verbally'.*
[72]; p224	*'the ability to quickly understand, approximate and manipulate numerical quantities'*
[56]; p452	*'Sense of approximate magnitudes'*
[51]; p125.	*'...system represents larger numerosities approximately'*
[60]	*This paper seems to use 'number sense' in the sense used by* [30]
[28]; p92.	*ANS: 'primitive cognitive system for making quantitative judgements and decisions: the ... ANS'*
[9]; p1	*'...approximate number sense that allows us to estimate quantity without the use of symbols and language.'*
[52]; p1	*'... an Approximate number system (ANS) that allows them [humans] to represent quantities as imprecise, noisy mental magnitudes without verbal counting or numerical symbols'*
[53]; p188	*'...an intuitive understanding of number. Without counting or the use of symbols, we are able to estimate, compare, and mentally manipulate large numerical quantities.'*
[69]; p83	*'an intuitive, non-symbolic, approximate sense of number that is available prior to the onset of schooling... The ANS represents numbers in a noisy imprecise fashion...'*

from another popular, much broader, definition of 'number sense' which defines the term as a core set of early numerical abilities which are crucial to acquire for later numerical development to be successful [25,30–33]. This broader definition of 'number sense' includes both non-symbolic manipulation and symbolic counting and arithmetic principles. It assumes that number sense involves 1) magnitude comparison; 2) object and verbal counting; 3) number identification; and 4) simple arithmetic. Here we only deal with the first definition of number sense or ANS. However, even a paper discussed here seems to blur the two definitions of number sense together [60].

Table 1 reviews the wide array of often *approximate* ANS definitions from the papers discussed here. Notably, several definitions provided do not necessitate an innate ANS and/or any special primitive representation of number. For example, the definition of [74] could be satisfied by manipulating symbolic numerical quantities in visuo-spatial working memory by some spatial addition or subtraction algorithm. However, as far as we understand this would be an unintended extension of the definition of 'number sense' and ANS. Some other definitions are similarly imprecise [28,72], with probably [9,52] giving the most clear and specific definitions.

In the following, we will discuss each published study which can be thought of as aiming to train the ANS with the intention of demonstrating carry over (transfer) effects to other mathematical abilities beyond non-symbolic number comparison (see Appendix A for the method of identifying these studies). When we refer to tables and figures in the current paper we just give simple table and figure numbers. In contrast, we will use the '#' symbol when we refer to tables and figures in the actually discussed paper (e.g. Fig. #7A means Fig. 7A in the paper under discussion and not in this paper).

2. Training with the Number Race software

Some studies used the so-called Number Race (NR) computer programme for training ANS (called 'number sense' or 'quantity representation' in these papers). For example, [72] states that 'The Number Race is an adaptive game designed to improve number sense.'. [74] says that they define 'number sense' in a narrow way, as the term is usually used in the cognitive neuroscience literature (p2; bottom right; see Table 1). They justify the creation of NR by arguing that dyscalculia ('a disorder in mathematical abilities', 'due to specific impairment in brain function'; p2; top left) is a 'core deficit in number sense' (p3.) and argue that NR was designed with this 'core deficit in mind' (p4.). Here, they state that NR aims to provide 'intensive training on numerical comparison' and to emphasize the 'links between numbers and space' (p4.). However, while a focus on a supposedly 'core deficit' would assume fairly specific training, NR is a mixed bag of training interventions which may affect many other cognitive skills and representations besides the ANS.

NR instruction is built on three domains [73]: First, it trains non-symbolic number comparison by prompting participants to choose between two groups of objects, one on the left and the other on the right. One of the two groups will have more objects than the other. For example, one group may have five objects while the other has three. There is also a timeline on the bottom of the screen with two characters, one for the player and the other representing the opponent. Whichever group the player chooses, the player's character will advance on the timeline the same number of spots as there were objects chosen and the opponent will automatically get the other group. So, if the player chooses the group with five objects his player will advance five spaces while the opponent would advance three. Since the first one to the finish line wins, it behoves the player to always try to choose the group with the most objects. The to-be-compared object arrays appear with varying levels of numerical distance between them, adapting to the comparison ability of the child. NR starts with easier number comparisons where there is large numerical distance between the to-be compared quantities and proceeds towards harder comparisons. The objects also appear in different sizes, either between or within groups. As will be shown below NR also aims to strengthen associations between spatial and numerical information. With regard to this, it is important to note that the ANS on its own is not supposed to include spatial elements, although this misconception is prevalent in the literature. In contrast, spatial-numerical associations seem culturally grounded [8], they appear gradually during development (e.g. [11,71]) and some researchers question whether they reflect properties of mature number representation at all, or they are rather related to working memory processes operating on representations [68].

A second domain that NR aims to train are links between various representations of number: non-symbolic representation, symbolic Arabic digits and aurally heard number words—primarily in that as the object arrays are shown, digits and aurally heard number words which correspond to the number of objects are also presented. This training domain goes well beyond the ANS: It constitutes both associative learning (linking representations) and training comparison operations with symbolic number representations. NR also presents the opportunity to practice a symbolic counting sequence. After the objects are transplanted from the top of the screen to the number line below, the narrator will name the spot which the player is at and then the avatar will jump a number of spaces to the new spot. While the spaces in between are not explicitly counted, the opportunity is there for the player to do so.

Third, NR also aims to increase the fluency of access to basic addition and subtraction facts. One way it does this is by stating the advancement of the player along the number line as an addition problem. For example, if the player is at spot 3 and chooses 5 objects, the programme will state, “Eight. Three plus five equals eight”. Sometimes the players will land on a trap. In this situation the programme will state the number of jumps back as a subtraction problem (e.g. “Oh no, you've landed on a trap. Eight minus two is six”.)