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High-rate quantization and transform coding with side information at the decoder ☆

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Abstract

We extend high-rate quantization theory to Wyner–Ziv coding, i.e., lossy source coding with side information at the decoder. Ideal Slepian–Wolf coders are assumed, thus rates are conditional entropies of quantization indices given the side information. This theory is applied to the analysis of orthonormal block transforms for Wyner–Ziv coding. A formula for the optimal rate allocation and an approximation to the optimal transform are derived. The case of noisy high-rate quantization and transform coding is included in our study, in which a noisy observation of source data is available at the encoder, but we are interested in estimating the unseen data at the decoder, with the help of side information.

We implement a transform-domain Wyner–Ziv video coder that encodes frames independently but decodes them conditionally. Experimental results show that using the discrete cosine transform results in a rate-distortion improvement with respect to the pixel-domain coder. Transform coders of noisy images for different communication constraints are compared. Experimental results show that the noisy Wyner–Ziv transform coder achieves a performance close to the case in which the side information is also available at the encoder.

Keywords: High-rate quantization; Transform coding; Side information; Wyner–Ziv coding; Distributed source coding; Noisy source coding

1. Introduction

Rate-distortion theory for distributed source coding [1–6] shows that under certain conditions, the performance of coders with side information available only at the decoder is close to the case in which both encoder and decoder have access to the side information. Under much more restrictive statistical conditions, this also holds for coding of noisy observations of unseen data [7,8].

One of the many applications of this result is reducing the complexity of video encoders by eliminating motion compensation, and decoding using past frames as side information, while keeping the efficiency close to that of motion-compensated encoding [9–11]. In addition, even if the image captured by the video encoder is corrupted by noise, we would still wish to recover the clean, unseen data

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at the decoder, with the help of side information, consisting of previously decoded frames, and perhaps some additional local noisy image.

In these examples, due to complexity constraints in the design of the encoder, or simply due to the unavailability of the side information at the encoder, conventional, joint denoising and coding techniques are not possible. We need practical systems for noisy source coding with decoder side information, capable of the rate-distortion performance predicted by information-theoretic studies. To this end, it is crucial to extend the building blocks of traditional source coding and denoising, such as lossless coding, quantization, transform coding and estimation, to distributed source coding.

!It was shown by Slepian and Wolf [3] that lossless distributed coding can achieve the same performance as joint coding. Soon after, Wyner and Ziv [4,12] established the rate-distortion limits for lossy coding with side information at the decoder, which we shall refer to as Wyner–Ziv (WZ) coding. Later, an upper bound on the rate loss due to the unavailability of the side information at the encoder was found in [5], which also proved that for power-difference distortion measures and smooth source probability distributions, this rate loss vanishes in the limit of small distortion. A similar high-resolution result was obtained in [13] for distributed coding of several sources *without* side information, also from an information-theoretic perspective, that is, for arbitrarily large dimension. In [14] (unpublished), it was shown that tessellating quantizers followed by Slepian–Wolf coders are asymptotically optimal in the limit of small distortion *and* large dimension.

It may be concluded from the proof of the converse to the WZ rate-distortion theorem [4] that there is no asymptotic loss in performance by considering block codes of sufficiently large length, which may be seen as vector quantizers, followed by fixed-length coders. This suggests a convenient implementation of WZ coders as quantizers, possibly preceded by transforms, followed by Slepian–Wolf coders, analogously to the implementation of nondistributed coders.

Practical distributed lossless coding schemes have been proposed, adapting channel coding techniques such as turbo codes and low-density parity-check codes, which are approaching the Slepian–Wolf bound, e.g., [15–23]. See [24] for a much more exhaustive list.

The first studies on quantizers for WZ coding were based on high-dimensional nested lattices [25–27], or heuristically designed scalar quantizers [16,28], often applied to Gaussian sources, with fixed-length coding or entropy coding of the quantization indices. A different approach was followed in [29–32], where the Lloyd algorithm [33] was generalized for a variety of settings. In particular, [32] considered the important case of ideal Slepian–Wolf coding of the quantization indices, at a rate equal to the conditional entropy given the side information. In [34–36], nested lattice quantizers and trellis-coded quantizers followed by Slepian–Wolf coders were used to implement WZ coders.

The Karhunen–Loève Transform (KLT) [37–39] for distributed source coding was investigated in [40,41], but it was assumed that the covariance matrix of the source vector given the side information does not depend on the values of the side information, and the study was not in the context of a practical coding scheme with quantizers for distributed source coding. Very recently, the distributed KLT was studied in the context of compression of Gaussian source data, assuming that the transformed coefficients are coded at the information-theoretic rate-distortion performance [42,43]. Most of the recent experimental work on WZ coding uses transforms [44,1,24].

There is extensive literature on source coding of a noisy observation of an unseen source. The nondistributed case was studied in [45–47], and [7,48–50,8] analyzed the distributed case from an information-theoretic point of view. Using Gaussian statistics and mean-squared error (MSE) as a distortion measure, [13] proved that distributed coding of two noisy observations *without* side information can be carried out with a performance close to that of joint coding and denoising, in the limit of small distortion *and* large dimension. Most of the operational work on distributed coding of noisy sources, that is, for a fixed dimension, deals with quantization design for a variety of settings [51–54], but does not consider the characterization of such quantizers at high rates or transforms.

A key aspect in the understanding of operational coding is undoubtedly the theoretic characterization of quantizers at high rates [55], which is also fundamental in the theoretic study of transforms for data compression [56]. In the literature reviewed at this point, the studies of high-rate distributed coding are information theoretic, thereby requiring arbitrarily large dimension, among other constraints. On the other hand, the aforementioned