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On Jacobi-type methods for blind equalization of paraunitary channels

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ABSTRACT

In this paper a study of the cumulant-based blind equalization algorithms PAJOD and PAFA is conducted. Both algorithms assume that the data have been pre-whitened and hence the problem reduces to the estimation of paraunitary channels. The main contribution of this paper is an efficient implementation of the PAJOD algorithm called PAJOD2. Second, a performance comparison between the PAJOD, PAJOD2 and PAFA algorithms is reported.

1. Introduction

Blind equalization of linear time-invariant MIMO channels refers to channel equalization techniques where only the observed signal is known. The observed signal is assumed to consist of an unknown convolutive mixture of input signals. The cumulant-based blind equalization algorithms Partial Approximate JOint Diagonalization (PAJOD) and PAraunitary FActorization (PAFA) were proposed in [1] and in [2], respectively. Both are based on contrast maximization and the working assumption for both algorithms is that the data have been pre-whitened. A method to perform pre-whitening has been proposed in [3].

Due to the pre-whitening the problem reduces to a search for a paraunitary equalizer. The PAFA algorithm looks for a paraunitary equalizer while the PAJOD algorithm only searches for a semi-unitary equalizer. The PAFA and PAJOD algorithms both consist of a Jacobi-type iteration where the Jacobi subproblem is solved by a computationally demanding resultant-based procedure. The PAFA algorithm requires the rooting of a 56th degree polynomial in each Jacobi subproblem. PAJOD requires the rooting of either a 3rd or 24th degree polynomial in each of its Jacobi subproblems, as will be explained later.

The main contribution of this paper is a more efficient implementation of the PAJOD algorithm called PAJOD2. Part of this work has been presented in [4]. The paraunitary equalizer PAFA fully takes the structure of the problem into account while the semi-unitary equalizer PAJOD only partially exploits the structure of the problem. However, the PAJOD algorithm is less computationally demanding than the PAFA algorithm. Hence, a comparison of the PAJOD, PAJOD2 and PAFA algorithms based on computer simulations will be reported.

The paper is organized as follows. First the notation used throughout the paper will be introduced. Since the algorithms are based on paraunitary filters and contrast optimization, a few basic notions about paraunitary filters and contrasts will be presented before discussing the system model. Next, in Section 2 a brief review of the PAJOD and PAFA algorithms is given. Furthermore, the more efficient implementation of the PAJOD algorithm called PAJOD2 will be presented. Section 3 will compare the PAJOD, PAJOD2 and PAFA methods based on computer

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simulations. We end the paper with a conclusion in Section 4.

1.1. Notation

Let $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{C}[z]$ denote the set of natural, integer, real, complex numbers and the set of polynomials in z with coefficients in $\mathbb{C}$, respectively. Furthermore, let $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^\dagger$, $\mathrm{Re}\{\cdot\}$, $\mathrm{Im}\{\cdot\}$ and $\|\cdot\|_F$ denote the conjugate, transpose, conjugate-transpose, pseudo-inverse, real part, imaginary part and the Frobenius norm of a matrix, respectively. The operator $\mathrm{diag}(\cdot)$ sets all the off-diagonal elements of a matrix equal to zero. Let $\mathbf{I}_R \in \mathbb{C}^{R\times R}$ denote the identity matrix. Given $\mathbf{A} \in \mathbb{C}^{m\times n}$, then $\mathbf{A}_{ij}$ denotes the ith row–jth column entry of $\mathbf{A}$. Finally, let $\mathbf{H}(z)=\sum_n \mathbf{H}(n)z^{-n}$.

1.2. Paraunitary filter

A filter $\mathbf{H}(z)=\sum_{l=0}^{L-1}\mathbf{H}(l)z^{-l} \in \mathbb{C}[z]^{R\times R}$ is paraunitary if $\mathbf{H}^H(1/z^*)\mathbf{H}(z)=\mathbf{I}_R$. The paraunitary filter $\mathbf{H}(z)$ satisfies the properties [5]:

- The inverse filter $\mathbf{H}^{-1}(z)=\mathbf{H}^H(1/z^*)$ is paraunitary.
- The channel impulse response matrix $\overline{\mathbf{H}}=[\mathbf{H}(0),\mathbf{H}(1),\ldots,\mathbf{H}(L-1)] \in \mathbb{C}^{R\times RL}$ is a semi-unitary matrix, i.e., $\overline{\mathbf{H}}\overline{\mathbf{H}}^H=\mathbf{I}_R$.

1.3. Contrast optimization

The notion of contrast optimization was introduced in [6] and applied in the framework of MIMO equalization in [7]. Let $\mathcal{H}$ and $\mathcal{S}$ be the set of paraunitary filters and the set of transmitted symbol sequences, respectively. Furthermore, let $\mathcal{H}\cdot\mathcal{S}$ denote the set of recovered symbol sequences and $\mathcal{T}$ denote the set of paraunitary equalizers that do not violate the working assumptions on $\mathcal{S}$ specified below. Moreover, let $\mathbf{I}$ denote the identity operator, then a function $\mathcal{J}(\mathbf{H};\mathbf{x})$ is called a contrast if it satisfies the properties [7]:

- *Invariance*: $\mathcal{J}(\mathbf{H};\mathbf{x})=\mathcal{J}(\mathbf{I};\mathbf{x}), \forall \mathbf{H}\in\mathcal{T}, \forall \mathbf{x}\in\mathcal{H}\cdot\mathcal{S}$.
- *Domination*: $\mathcal{J}(\mathbf{H};\mathbf{x})\le\mathcal{J}(\mathbf{I};\mathbf{x}), \forall \mathbf{H}\in\mathcal{H}, \forall \mathbf{x}\in\mathcal{S}$.
- *Discrimination*: $\mathcal{J}(\mathbf{H};\mathbf{x})=\mathcal{J}(\mathbf{I};\mathbf{x}), \forall \mathbf{x}\in\mathcal{S} \Rightarrow \mathbf{H}\in\mathcal{T}$.

Under the assumption that there exists an equalizer that will fully recover the symbols, an equalizer corresponding to the global maximum of the contrast function is guaranteed to recover the symbol sequence.

1.3.1. System model

Let $\mathbf{s}(n), \mathbf{x}(n) \in \mathbb{C}^R$ be the symbol and observation vector at time instant $n\in\mathbb{N}$, respectively. Assume that $\mathbf{s}(n)$ and $\mathbf{x}(n)$ are related via

$$\mathbf{x}(n)=\sum_{k=0}^{K-1}\mathbf{F}(k)\mathbf{s}(n-k),$$

where $\mathbf{F}(k)\in\mathbb{C}^{R\times R}$, $k\in[0,K-1]$, is the channel impulse response of the paraunitary filter $\mathbf{F}(z)$. The problem is to estimate the symbol sequence $\{\mathbf{s}(n)\}$ from the observation sequence $\{\mathbf{x}(n)\}$. This is done by the equalizer $\mathbf{H}(z)$ such that

$$\mathbf{y}(n)=\sum_{l=0}^{L-1}\mathbf{H}(l)\mathbf{x}(n-l)=\sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\mathbf{H}(l)\mathbf{F}(k)\mathbf{s}(n-l-k),$$

where $\mathbf{H}(l)\in\mathbb{C}^{R\times R}$, $l\in[0,L-1]$, are the channel impulse response coefficients of $\mathbf{H}(z)$ and $\mathbf{y}(n)$ is the recovered symbol vector at time instant n. Since $\mathbf{F}(z)$ is paraunitary, we know that the equalizer $\mathbf{H}(z)$ is also paraunitary and that $K=L$. In the PAJOD and PAFA methods, the working assumptions on the transmitted signal sequences are:

- $\mathbf{s}_r(n)$ are mutually independent i.i.d., zero-mean processes with unit-variance for all $r\in[1,R]$.
- $\mathbf{s}(n)$ is stationary up to order 4 and hence the marginal cumulants of order 4 do not depend on n.
- At most one source has zero marginal cumulant of order 4.

2. Paraunitary equalization algorithms

The PAJOD and PAFA blind paraunitary equalization algorithms will be briefly reviewed and the computationally improved version of PAJOD algorithm called PAJOD2 will be introduced.

2.1. PAJOD

In [1], the cumulants of the observed data are stored in a set of $RL\times RL$ matrices $\mathbf{M}(\mathbf{b},\gamma)$ in such a way that for a fixed pair $(\mathbf{b},\gamma)=([b_1,b_2],[\gamma_1,\gamma_2])$ we have the relation

$$\mathbf{M}_{\alpha_1R+a_1,\alpha_2R+a_2}(\mathbf{b},\gamma)=\mathrm{Cum}[y_{a_1}(n-\alpha_1),y^*_{a_2}(n-\alpha_2),y_{b_1}(n-\gamma_1),y^*_{b_2}(n-\gamma_2)].$$

Moreover, in [1] it is shown that the function

$$\mathcal{J}_2^2=\sum_{\mathbf{b},\gamma}\|\mathrm{diag}(\overline{\mathbf{H}}\mathbf{M}(\mathbf{b},\gamma)\overline{\mathbf{H}}^H)\|_F^2 \tag{1}$$

is a contrast function, where $\|\mathrm{diag}(\mathbf{A})\|_F^2=\sum_i|\mathbf{A}_{ii}|^2$ and $\overline{\mathbf{H}}=[\mathbf{H}(0),\mathbf{H}(1),\ldots,\mathbf{H}(L-1)]\in\mathbb{C}^{R\times RL}$. Due to the paraunitary assumption on $\mathbf{H}(z)$, the matrix $\overline{\mathbf{H}}$ is a semi-unitary matrix, i.e., $\overline{\mathbf{H}}\overline{\mathbf{H}}^H=\mathbf{I}_R$.

2.1.1. Jacobi procedure for semi-unitary matrices

To numerically find the semi-unitary matrix $\overline{\mathbf{H}}$ that will maximize the contrast (1) a Jacobi procedure was proposed in [1]. This procedure can be seen as a double extension of the JADE algorithm [8,9]. First, the unknown matrix is semi-unitary instead of unitary. Second, only the R first diagonal entries are of interest.

A Jacobi procedure is based on the fact that any $RL\times RL$ unitary matrix with determinant equal to one can be parametrized as a product of Givens rotations [10]:

$$\mathbf{V}=\prod_{p=1}^{RL-1}\prod_{q=p+1}^{RL}\Theta[p,q]^H,$$

where $\Theta[p,q]$ is equal to the identity matrix, except for entries

$$\Theta_{pp}[p,q]=\Theta_{qq}[p,q]=\cos(\theta[p,q]),$$

$$\Theta_{qp}[p,q]=-\Theta_{pq}[p,q]^*=\sin(\theta[p,q])e^{i\phi[p,q]},\theta[p,q],\phi[p,q]\in\mathbb{R}.$$