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Blind source separation of convolutive mixtures of non-circular linearly modulated signals with unknown baud rates

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ABSTRACT

This paper addresses the problem of blind separation of convolutive mixtures of BPSK and circular linearly modulated signals with unknown (and possibly different) baud rates and carrier frequencies. In previous works, we established that the Constant Modulus Algorithm (CMA) is able to extract a source from a convolutive mixture of circular linearly modulated signals. We extend the analysis of the extraction capabilities of the CMA when the mixing also contains BPSK signals. We prove that if the various source signals do not share any non-zero cyclic frequency nor any non-conjugate cyclic frequencies, the local minima of the constant modulus cost function are separating filters. Unfortunately, the minimization of the Godard cost function generally fails when considering BPSK signals that have the same rates and the same carrier frequencies. This failure is due to the existence of non-separating local minima of the Godard cost function. In order to achieve the separation, we propose a simple modification of the Godard cost function which only requires knowledge of the BPSK sources frequency offsets at the receiver side. We provide various simulations of realistic digital communications scenarios that support our theoretical statements.

1. Introduction

The blind source separation of convolutive mixtures of linearly modulated signals has mainly been studied in the case where the signals share the same known baud rate, and when the sampling frequency of the multivariate received signal coincides with this baud-rate. In this context, to be referred to in the sequel as the *stationary case*, the discrete-time received signal coincides with the output of an unknown MIMO filter driven by the sequences of symbols sent by the various transmitters. In most cases, these sequences are independent and identically distributed, and several methods have been proposed in order to extract each of them from the observation (see e.g. [3,7,8,10,11]). The source separation problems that are encountered in the context of passive listening are however more complicated because the transmitters are usually completely unknown to the receiver, and have no reason to transmit linearly modulated signals sharing the same baud-rates. It is therefore quite relevant to address the problem of blind separation of linearly modulated signals with unknown, and possibly different, baud rates. In this context, the received signal is sampled at any frequency satisfying the Shannon sampling theorem, so that the corresponding discrete-time signal is cyclostationary with unknown cyclic frequencies. If the cyclic frequencies were known at the receiver side, it would be easy to generalize the usual blind source separation approaches based on the optimization of contrast functions depending on higher order cumulants. However, when the cyclic frequencies are unknown, it is impossible to consistently estimate the cumulants, a conceptual problem first remarked by Ferreol and Chevalier ([6]) in the context of blind separation of instantaneous mixtures.

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An obvious approach would consist in estimating the unknown cyclic frequencies. However, this is a difficult task if the excess bandwidths of the transmitted signals are low and if the duration of observation is not large enough.

In contrast with the cumulants, the constant modulus cost function can be consistently estimated in the cyclo-stationary context. In [9], we considered only source signals that transmit second-order circular symbol sequences, and we have shown that in this case, to be referred to as the *circular case*, the minimization of the Godard cost function allows to extract the sources using a deflation approach if their baud-rates are different one from another. If certain baud rates coincide, sufficient conditions for the separation have been established in [9]. Although we have not been able to prove that separation is achieved in the most general case, all the simulations we have performed strongly suggest that the minimization of the Godard cost function is successful in the circular case. The purpose of this paper is to address this issue when in the non-circular source signals, which will be referred to as the *non-circular case*, and to show how the separation method based on the minimization of the CMA contrast function coupled with a deflation approach can be adapted to this context. As in [9] we only focus in this paper on the separation of the first source.

In order to simplify the presentation of our results, we only consider the case where the non-circular signals are BPSK signals. We begin by defining in Section 2 the context of our study and giving a brief description of the considered signals and criteria. In Section 3 we prove that the Godard cost function is still successful if the sources do not share the same baud rates and the same carrier frequencies. We also prove, in Section 4, that contrary to the circular case, the minimization of the Godard cost function fails to separate two BPSK signals sharing the same baud rate and the same carrier frequency. We show that this is due to the existence of non-separating local minima of the Godard cost function, toward which the minimization algorithms seem to converge quite often. We also show that it is possible to modify the Godard cost function in order to achieve source separation of K non-circular BPSK modulated signals sharing the same known (or well estimated) carrier frequency. Section 5 briefly generalizes this result to more general mixtures. The new modified CMA algorithm needs the estimation of the carrier frequencies offsets of the non-circular source signals, or equivalently the estimation of the "significant" non-conjugate cyclic frequencies of the received signal. Fortunately, this is a much easier task than the estimation of baud rates, because the non-conjugate cyclic correlation coefficients of the received signal at twice the frequency offsets are not affected by possible low excess bandwidths of the source signals (see [1]). Numerical results are finally presented in Section 6.

Notations: If $(u_n)_{n\in\mathbb{Z}}$ is a discrete-time sequence, we denote by $\langle u_n \rangle$ the time average operator defined as

$$\langle u_n \rangle = \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} u_n$$

If x is a complex valued random variable, we denote by $c_4(x)$ its fourth order cumulant defined by $cum\{x,x^*,x,x^*\}$. If $(x(n))_{n\in\mathbb{Z}}$ is a discrete-time cyclostationary sequence, we define, when it makes sense, the cyclo-correlation at cyclic-frequency α and time lag m

$$\forall \alpha \in \left(-\frac{1}{2},\frac{1}{2}\right],\ \forall m \in \mathbb{Z},\quad R_x^{(\alpha)}(m) = \langle \mathbb{E}(x(n+m)x(n)^* e^{-2i\pi n\alpha}) \rangle$$

and the non-conjugate cyclo-correlation at cyclic-frequency α_c and time lag m

$$\forall \alpha_c \in \left(-\frac{1}{2},\frac{1}{2}\right],\ \forall m \in \mathbb{Z},\quad R_{c,x}^{(\alpha_c)}(m) = \langle \mathbb{E}(x(n+m)x(n) e^{-2i\pi n\alpha_c}) \rangle$$

For a wide-sense cyclostationary continuous-time random process $(x_a(t))_{t\in\mathbb{R}}$ we denote by $R_{a,x}^{(\alpha_a)}(\tau)$ and by $R_{a,c,x}^{(\alpha_{a,c})}(\tau)$ the cyclic correlation coefficient and respectively non-conjugate cyclic correlation coefficient at cyclic-frequency α_a (respectively non-conjugate cyclic frequency $\alpha_{a,c}$) and time lag τ.

For an interval $\mathcal{B}$, we denote by $\mathcal{F}(\mathcal{B})$ the set of all functions $f_a(t) \in \mathbb{L}^2(\mathbb{R})$ such that

$$f_a(t) = \int_{\mathcal{B}} s^{2i\pi\nu t} \hat{f}_a(\nu)\, d\nu$$

In other words, a square integrable function f_a is an element of $\mathcal{F}(\mathcal{B})$ if and only if its Fourier transform $\hat{f}_a(\nu)$ is zero outside $\mathcal{B}$.

2. Problem statement

2.1. Assumptions

We assume that K unknown transmitters send linearly modulated signals sharing the same frequency bandwidth. The receiver is equipped with a sensor of N-arrays, and the corresponding N-dimensional received signal is sampled at rate $1/T_e$ supposed to satisfy the Shannon sampling theorem. For any k, $k=1,\ldots,K$, the signal transmitted by source k is obtained by linearly modulating a unit variance zero mean i.i.d. sequence of symbols $\{a_{k,n}\}_{n\in\mathbb{Z}}$ with a shaping filter $g_{a,k}$

$$s_{a,k}(t) = \sum_{n\in\mathbb{Z}} a_{k,n} g_{a,k}(t-nT_k)$$

We denote by T_k the symbol period of the source number k and we consider a shaping filter of limited bandwidth $[-(1+\gamma_k)/2T_k,(1+\gamma_k)/2T_k]$, where γ_k is the excess bandwidth factor, belonging to $[0,1)$. The bandwidth of the complex envelope of transmitted signal k is then $[-(1+\gamma_k)/2T_k,(1+\gamma_k)/2T_k]$.

In order to simplify the presentation of the results we make the following assumption:

- the symbol sequence $\{a_{k,n}\}_{n\in\mathbb{Z}}$ is either second order circular or corresponding to a BPSK constellation (i.e. equal to ± 1) for each k.

The propagation channels between each transmitter and the receiver are assumed to be frequency selective. Moreover, the carrier frequencies of the various transmitted signals of course do not coincide with the center frequency of the receive filter of the receiver. Hence, the contribution of each transmitted signal at the receiver