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A study on image denoising in contourlet domain using the alpha-stable family of distributions

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ABSTRACT

In the past decade, several image denoising techniques have been developed aiming at recovering signals from noisy data as much as possible along with preserving the features of an image. This paper proposes a new image denoising method in the contourlet domain by using the alpha-stable family of distributions as a prior for contourlet image coefficients. The univariate symmetric alpha-stable distribution (SαS) is mostly suited for modeling of the i.i.d. contourlet coefficients with high non-Gaussian property and heavy tails. In addition, the bivariate SαS exploits the dependencies between the coefficients across scales. In this paper, using the univariate and bivariate priors, Bayesian minimum mean absolute error and maximum a posteriori estimators are developed in order to estimate the noise-free contourlet coefficients. To estimate the parameters of the alpha-stable distribution, a spatially-adaptive method using fractional lower order moments is proposed. It is shown that the proposed parameter estimation method is superior to the maximum likelihood method. An extension to color image denoising is also developed. Experiments are carried out using noise-free images corrupted by additive Gaussian noise, and the results show that the proposed denoising method outperforms other existing methods in terms of the peak signal-to-noise ratio and mean structural similarity index, as well as in visual quality of the denoised images.

1. Introduction

Denoising is a problem of estimating the noise-free image from noisy observations while preserving the image features. The image denoising techniques may be classified into spatial [1,2], and transform domain [3–7] approaches. The image denoising in the transform domain has attracted considerable interest in view of its improved performance at recovering signals from noisy data. In the transform domain approach, denoising process is performed on the transformed coefficients of different transforms such as wavelet transform [5,6]. In fact, the wavelet shrinkage method, proposed by Donoho [7], is the most demonstrative one in which a simple and non-probabilistic thresholding is used to remove noise from an image. However, it is known that the wavelet transform is good at isolating discontinuities at edge points and cannot efficiently capture the smoothness along the contour [8,9]. In addition, applying wavelet to an image results in capturing limited directional information. In [10], the principal component analysis has been proposed to overcome the drawbacks of the wavelet transform in highly-structured images. However, these components are highly affected by the noise. In [11], the K-SVD algorithm has been proposed for the same purpose. However, exhaustive search in learned dictionaries gives rise to a time-consuming algorithm. Another class of image denoising techniques is the non-local means (NLM) algorithms [12–18]. The NLM algorithms estimate a pixel by a weighted average of the local and non-local pixels throughout the image and perform denoising by exploiting the natural redundancy of the patterns inside an image. In [19], similar to motion estimation algorithms, a block-wise matching has been used to preprocess the noisy image followed by a transform domain shrinkage, known as BM3D. However, the accuracy of such block correlations is highly dependent on the noise. In [20], a patch-based locally-optimal Wiener filter has been proposed for image denoising. This method uses similar patches to estimate the filter parameters. In [21], a spatially adaptive iterative singular-value thresholding method has been proposed, which provides slightly better performance in terms of peak signal-to-noise ratio (PSNR) than that provided by BM3D.

To enhance the sparsity and effectively capture the directional information in natural images, other multi-scale and multi-resolutional transforms, such as wavelet-packets [22], complex wavelet [23–25], curvelet [26], or contourlet [8,9,27–29] transforms, have been proposed. The better sparseness and decorrelation properties of

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these transforms result in improved image denoising schemes. In [24,30], the image denoising is performed in complex wavelet domain, which provides more directionality than that provided by wavelet, yet is not efficient to handle 2-D singularities. In [26], the curvelet domain image denoising has been proposed. The curvelet transform provides higher directional information of an image resulting in a denoising scheme with visually improved image and more edge preservation. However, the curvelet transform has originally been defined on concentric circles in the continuous domain and the process of discretization is complex and time-consuming. Therefore, to overcome these disadvantages of the curvelet transform, the contourlet transform has been proposed in [9].

The contourlet transform provides not only the multiscale and time-frequency localization features of the wavelet transform, but also offers a higher degree of directionality with better sparseness. In view of this, it has been shown in [9] that image denoising in the contourlet domain is superior to that in the wavelet domain. Most of the image denoising algorithms in the contourlet domain have been developed based on the thresholding or shrinkage functions [9,27], in which the coefficients with small magnitudes are simply set to zero, while the rest are kept unchanged in the case of hard-thresholding, and shrunk in the case of soft-thresholding.

In recent years, statistical models have been adopted for the transform domain coefficients in which the image and noise are modeled as random fields and Bayesian methods are employed to develop shrinkage functions for estimation of the noise-free coefficients from the noisy observations. It is to be noted that the prior distributions for the original image and the noise have considerable effect on the performance of the denoising process. Several prior distributions have been employed to characterize the transform coefficient properties such as their sparsity, i.e., having a large number of small coefficients along with a small number of large coefficients [5,6,25,31–38]. The contourlet coefficients have been shown to be highly non-Gaussian [9,39–41], i.e., having large peaks around zero and tails heavier than that of a Gaussian probability density function (PDF). In view of this, the contourlet coefficients have been modeled formerly by the generalized Gaussian distribution [9].

Through modeling of the actual data, we have shown in [40,41], that the contourlet-domain subband decomposition of real images has significant non-Gaussian statistics that are best described by families of heavy-tailed distributions, such as the alpha-stable family. Motivated by the modeling results, in this work, we propose a new image denoising technique in the contourlet domain based on the alpha-stable family of distributions as a prior for the contourlet coefficients. We will derive the Bayesian minimum mean absolute error (MMAE) and maximum a posteriori (MAP) estimators using the alpha-stable distribution to obtain the noise-free contourlet coefficients. We first assume that the contourlet coefficients are independent and identically distributed by the univariate alpha-stable distribution. Then, we consider the across-scale dependencies of the contourlet coefficients by employing the bivariate alpha-stable distribution to capture these dependencies. In order to estimate the parameters of the model, we propose a spatially-adaptive method based on fractional lower order moments. An extension to color image denoising will also be developed. Several experiments are conducted to evaluate the performance of the proposed denoising scheme and to compare it with those of the current state-of-the-art techniques. The estimated images are compared with the original ones in terms of the PSNR and mean structural similarity (MSSIM) index, as well as in visual quality of the denoised images.

The paper is organized as follows: Section 2 presents briefly the contourlet transform. In Section 3, the alpha-stable distribution and results on modeling the contourlet coefficients of images using this distribution are presented. In Section 4, the image denoising algorithm based on either the MMAE or MAP estimator is presented. In Section 5, the performance of the proposed algorithms is examined and compared to those of the other existing methods. Section 6 concludes the paper.

2. The contourlet transform

The contourlet transform, a new image decomposition scheme proposed in [9], provides an efficient representation for two-dimensional signals with smooth contours and in this case, outperforms the wavelet transform, which fails to recognize the smoothness of the contour. The contourlet transform also has the multiscale and time-frequency localization features of the wavelet transform [42]. In addition, it offers a higher degree of directionality with better sparseness. Further, in view of the use of iterated filter banks, it is computationally efficient [9]. There are number of other structures, such as the complex wavelet [23], ridgelet [43,44] and curvelet [45,46], that also provide multiscale and directional image representation. However, most of these structures are not flexible in the sense that one cannot use different number of directions at each scale. Moreover, since the contourlet transform has been introduced in the discrete domain, it overcomes the blocking artifact deficiency of the curvelet transform (Fig. 1). It should be noted that the use of downsamplers and upsamplers in the structure of the contourlet transform makes it shift-variant, which may produce artifacts around the singularities, e.g., edges. Hence, the cycle spinning method [27,47] is employed to compensate for the lack of translation invariance. It is a simple, yet efficient, method to improve the denoising performance for a shift-variant transform. In fact, the cycle spinning is to average out the translation dependence of the subsampled contourlet transform and can be expressed as

$$\hat{I} = \frac{1}{M \times N} \sum_{m=1}^{M} \sum_{n=1}^{N} \left(ICT\,(S_{-m,-n}(h(CT\,(S_{m,n}(I))))\right) \tag{1}$$

in which I and $\hat{I}$ are noisy and denoised images, CT and ICT are the contourlet transform and its inverse, respectively, $S_{m,n}$ is the cycle spinning operator with (m,n) as shifts in the horizontal and vertical directions, and h is the denoising operator in the contourlet domain [47].

3. Modeling of contourlet coefficients using the alpha-stable distribution

In order to model the contourlet subband coefficients of an image, we propose the use of $S\alpha S$ distribution as a prior for the contourlet coefficients of a noisy image. A random variable $X \sim S_\alpha(\gamma, \beta, \delta)$ with univariate alpha-stable distribution is described by its characteristic function given by [48]

$$\Phi_{\alpha,\gamma,\beta,\delta}(\omega) = \exp\left\{ j\delta\omega - \gamma|\omega|^\alpha [1 + j\beta\ \mathrm{sign}(\omega)\varpi(\omega, \alpha)]\right\} \tag{2}$$

where

$$\varpi(\omega, \alpha) = \begin{cases} \tan\dfrac{\alpha\pi}{2} & \text{if } \alpha \neq 1 \\ \dfrac{2}{\pi}\log|\omega| & \text{if } \alpha = 1 \end{cases} \tag{3}$$

and α is a characteristic exponent, $(0 < \alpha \le 2)$, $\beta \in [-1, 1]$ is a skewness parameter, $\delta \in \Re$ is a location parameter and $\gamma > 0$ a dispersion parameter. For a particular class of the alpha-stable distributions, called the standard symmetric alpha-stable (SαS) distribution, $\delta = \beta = 0$. The characteristic exponent α is the most important parameter in determining the shape of the distribution [48,49]. The smaller the value of α, the heavier the tail of the distribution. This implies that random variables following the SαS distribution with small characteristic exponents are highly impulsive. In