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Short Communication

Eigenspace-based beamforming technique for multipath coherent signals reception

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ABSTRACT

A novel eigenspace-based beamforming technique is presented for receiving multipath coherent signals in the presence of uncorrelated interferences. Firstly, a subaperture minimum variance distortionless response beamformer based on spatial smoothing is applied, and then its weights are projected onto the signal-plus-interference subspace of the full-aperture data covariance matrix to obtain the multipath signals combination beamformer. Theoretical analysis has proved the proposed method can effectively get the coherent signals combination gain without reducing array aperture. Due to the employment of eigenspace, our method requires less prior information and converges faster than existing solutions. Simulation examples illustrate the effectiveness of the proposed technique.

1. Introduction

Adaptive beamforming techniques have been widely studied during the past decades [1]. The objective of beamforming is to adaptively enhance signals from desired directions and suppress interferences from other directions using an array of sensors. Conventional adaptive beamforming methods suffer from signal cancellation in the presence of coherent or highly correlated signals, which is a common case in practice due to multipath propagation.

To avoid such signal cancellation, a variety of techniques have been introduced [2]. Among currently available solutions, spatial smoothing for regular arrays and its improved versions [3,4] are regarded as the most promising ones. These methods decorrelate coherent signals prior to beamforming by dividing array into subarrays and averaging the data covariance matrices of subarrays. After spatial smoothing, a subaperture minimum variance distortionless response (MVDR) beamformer can be adopted to receive the coherent signal from main-lobe direction. The limitation of spatial smoothing is that effective aperture of an array would be significantly reduced. Another inevitable disadvantage is, all these approaches tried to nullify sidelobe coherent signals to prevent them from canceling the main-lobe signal. However, a truly effective beamformer should constructively combine coherent signals instead of canceling all but one of them.

Given the direction of arrival (DOA) information of all incident signals, some beamforming methods estimate the complex amplitude of each coherent signal component and impose multiple constraints on the array for multipath signals combination [5]. The famous Rake system based on pilots realizes it by evaluating the delay of each path. In practice, to obtain exact DOAs of all coherent signals is computationally expensive and pilot symbols are generally unavailable either. Blind beamformers have been introduced in [6], but such methods converge slowly and require desired coherent signals and interferences to have different statistical properties.

In this paper, a novel beamformer for receiving multipath coherent signals with uncorrelated interferences is presented. It is designated as eigenspace-based beamformer (ESB) since its key step is to project weight vector of the subaperture MVDR beamformer with spatial smoothing onto the signal-plus-interference subspace of full-aperture data covariance matrix. As shown in simulation results, the proposed beamformer inherits robustness and rapid convergence rate from conventional eigenspace-based algorithms. Moreover, it can get both multipath signals combination gain and full array aperture gain with less prior information.

Throughout the paper, bold lowercase letters are used for vectors, and bold uppercase letters are for matrices. Superscripts T, H, and $*$ represent transpose, conjugate transpose, and complex conjugate respectively. $\|\cdot\|$ symbolizes the Euclidean norm and $|\cdot|$ represents absolute value. $0_{M\times N}$ is $M\times N$ zero matrix. $\boldsymbol{I}$ denotes identity matrix.

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2. Signal model and optimal beamforming technique

2.1. Data modeling

Assume that a group of P fully coherent signals from directions θ_p,p=1,2…P and Q uncorrelated interferences from directions φ_q,q=1, 2…Q impinge on a uniform linear array of M omnidirectional sensors. All the sources are assumed to be narrow-band signals with the same center frequency. The $M\times 1$ received data vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{x}(t) = s_d(t)\sum_{p=1}^{P}\alpha_p\boldsymbol{a}(\theta_p) + \sum_{q=1}^{Q}\boldsymbol{a}(\varphi_q)s_q(t) + \boldsymbol{n}(t) \ , \tag{1}$$

where $s_d(t)$ represents the desired signal waveform with power σ_d^2 and α_p denotes complex amplitude of the pth coherent signal. $\boldsymbol{a}(\theta)$ is the steering vector to direction θ. $s_q(t)$ represents the waveform of the qth interference. $\boldsymbol{n}(t)$ denotes additive noise vector whose element is with zero mean and variance σ_n^2. Eq. (1) can be rewritten in vector form as

$$\boldsymbol{x}(t) = \boldsymbol{a}_d s_d(t) + \boldsymbol{A_I}\boldsymbol{s_I}(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \ , \tag{2}$$

where $\boldsymbol{a}_d=\sum_{p=1}^{P}\alpha_p a(\theta_p)$ denotes the steering vector of $s_d(t)$ and the response matrix of sources $A=[a_d, A_I]$and $A_I=[a(\varphi_1)\cdots a(\varphi_Q)]$. The source vector $s^T(t)=[s_d(t), s_I^T(t)]$, $s_I^T(t)=[s_1(t)\ldots s_Q(t)]$. Then the covariance matrix of $\boldsymbol{x}(t)$ is

$$\begin{aligned}\boldsymbol{R}_x &= E\left[\boldsymbol{x}(t)\boldsymbol{x}^H(t)\right]\\ &= \sigma_d^2\boldsymbol{a}_d\boldsymbol{a}_d^H + \boldsymbol{A}_I\boldsymbol{R}_I\boldsymbol{A}_I^H + \sigma_n^2\boldsymbol{I}\\ &= \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I} \ ,\end{aligned} \tag{3}$$

where $\mathbf{R}_s=E[s(t)s^H(t)]$and $\mathbf{R}_I=E[s_I(t)s_I^H(t)]$represent the covariance matrix of sources and interferences respectively.

2.2. Optimal beamformers

Given $\boldsymbol{x}(t)$, α_p and $\boldsymbol{a}(\theta_p)$, the optimal beamformer with maximum output signal-to-interference-plus-noise ratio (SINR) can be directly achieved by the MVDR method. The corresponding weight vector is determined as

$$\boldsymbol{w}_{MVDR} = \boldsymbol{R}_{I+n}^{-1}\boldsymbol{a}_d/\left(\boldsymbol{a}_d^H\boldsymbol{R}_{I+n}^{-1}\boldsymbol{a}_d\right) \ , \tag{4}$$

where $\mathbf{R}_{I+n}=\mathbf{A}_I\mathbf{R}_I\mathbf{A}_I^H+\sigma_n^2 I$. But usually in practice, only the steering vector of one signal is available, e.g. $\boldsymbol{a}(\theta_1)$. If we use MVDR beamformer directly by taking $\boldsymbol{a}(\theta_1)$ as the desired steering vector, significant signal cancellation would occur [2].

The minimum mean square error (MMSE) beamformer based on $s_d(t)$ is characterized by the capability to combine coherent signals as well ([7], Ch.6.12). It has been proved in [7] that weights of the MMSE beamformer is of the same form as $\boldsymbol{w}_{MVDR}$ given in (4) statistically.

Next, we briefly demonstrate the steady performance of $\boldsymbol{w}_{MVDR}$ in terms of the output SINR. Using matrix inversion lemma, the inversion of $\mathbf{R}_{I+n}$ yields

$$\boldsymbol{R}_{I+n}^{-1} = \left(\boldsymbol{A}_I\boldsymbol{R}_I\boldsymbol{A}_I^H + \sigma_n^2\boldsymbol{I}\right)^{-1} = \frac{1}{\sigma_n^2}\left(\boldsymbol{I} - \boldsymbol{A}_I\left(\sigma_n^2\boldsymbol{R}_I^{-1} + \boldsymbol{A}_I^H\boldsymbol{A}_I\right)^{-1}\boldsymbol{A}_I^H\right) \tag{5}$$

With uncorrelated interferences, $\boldsymbol{R}_I$ is the diagonal matrix with interference power as its diagonal entry. Meanwhile diagonal entries of $\mathbf{A}_I^H\mathbf{A}_I$ are all M, since $\left\|a(\varphi_q)\right\|=\sqrt{M}$ [7]. It is clear that only the interference whose power is considerably greater than σ_n^2/M affects signal reception. Then we just take such "strong" interferences into account in the sequel, so $\sigma_n^2\boldsymbol{R}_I^{-1}$ can be neglected compared with $\mathbf{A}_I^H\mathbf{A}_I$.

Denoting $\boldsymbol{P}_I^{\perp} = \boldsymbol{I} - \boldsymbol{A}_I(\boldsymbol{A}_I^H\boldsymbol{A}_I)^{-1}\boldsymbol{A}_I^H$ as the orthogonal subspace of interferences, we can deduce from (4) and (5) that

$$\boldsymbol{w}_{MVDR} \approx \frac{\left(\boldsymbol{I} - \boldsymbol{A}_I\left(\boldsymbol{A}_I^H\boldsymbol{A}_I\right)^{-1}\boldsymbol{A}_I^H\right)\boldsymbol{a}_d}{\boldsymbol{a}_d^H\left(\boldsymbol{I} - \boldsymbol{A}_I\left(\boldsymbol{A}_I^H\boldsymbol{A}_I\right)^{-1}\boldsymbol{A}_I^H\right)\boldsymbol{a}_d} = \frac{\boldsymbol{P}_I^{\perp}\boldsymbol{a}_d}{\boldsymbol{a}_d^H\boldsymbol{P}_I^{\perp}\boldsymbol{a}_d} \tag{6}$$

Eq. (6) indicates that strong interferences can be effectively nullified by $\boldsymbol{w}_{MVDR}$. Indeed it has been further demonstrated in [7] that, the MVDR beamforming technique can perfectly nullify strong sidelobe jammers regardless of the power of other sources. Then the output SINR of optimal beamformers is given by

$$SINR_{opt} = \frac{P_s}{P_n + P_I} \approx \frac{E\left[\left|s_d(t)\boldsymbol{w}_{MVDR}^H\boldsymbol{a}_d\right|^2\right]}{E\left[\left|\boldsymbol{w}_{MVDR}^H\boldsymbol{n}(t)\right|^2\right]} = \frac{\sigma_d^2\left|\boldsymbol{w}_{MVDR}^H\boldsymbol{a}_d\right|^2}{\sigma_n^2\|\boldsymbol{w}_{MVDR}\|^2}, \tag{7}$$

where P_s, P_n and P_I denote the output power of signal, noise and interferences respectively. Substituting (6) into (7) leads to

$$SINR_{opt} = \frac{\sigma_d^2\left|\boldsymbol{a}_d^H\boldsymbol{P}_I^{\perp}\boldsymbol{a}_d\right|^2}{\sigma_n^2\|\boldsymbol{P}_I^{\perp}\boldsymbol{a}_d\|^2} = \frac{\sigma_d^2}{\sigma_n^2}\boldsymbol{a}_d^H\boldsymbol{P}_I^{\perp}\boldsymbol{a}_d \tag{8}$$

From (8), we find the output SINR of optimal beamformers is matched to the composite steering vector $\boldsymbol{a}_d$, which is associated with all the desired coherent signals.

3. Eigenspace-based beamforming technique

Without $\boldsymbol{a}_d$ or $s_d(t)$, spatial smoothing can be adopted to enable the conventional MVDR beamformer based on $\boldsymbol{a}(\theta_1)$ [3]. The method divides original array into L subarrays in such a way that the lth subarray data vector is $\boldsymbol{x}_l(t) = [x_l(t), x_{l+1}(t), \cdots, x_{l+N-1}(t)]^T$, $l = 1, \ \cdots \ M-N+1$, where N is the number of subarray sensors. Taking forward spatial smoothing as an instance, the MVDR beamformer with spatial smoothing (designated as SSB) has the following form [3]

$$\boldsymbol{w}_{SSB} = \boldsymbol{R}_{smooth}^{-1}\boldsymbol{a}_1(\theta_1)/\left(\boldsymbol{a}_1^H(\theta_1)\boldsymbol{R}_{smooth}^{-1}\boldsymbol{a}_1(\theta_1)\right), \tag{9}$$

where $R_{smooth}=\sum_{l=1}^{L}E[\boldsymbol{x}_l(t)\boldsymbol{x}_l^H(t)]/L$. $\boldsymbol{a}_1(\theta_1)$ is a $N\times 1$ column vector denoting steering vector of the first subarray. $\boldsymbol{w}_{SSB}$ cannot be satisfied as it merely receives the coherent source from θ_1, and the spatial resolution capability of array will decreases due to the use of subarray aperture.

However, it is worth noting that $\boldsymbol{w}_{SSB}$ given in (9) can reject sidelobe signals with the mainlobe signal remained [3]. Meanwhile as stated in Section 2.2, sidelobe strong jammers can be nullified by the MVDR beamforming operation perfectly. With this in mind, we adopt MATLAB notation $\boldsymbol{A}(1{:}\ N, :\ )$ to denote the submatrix of $\boldsymbol{A}$ formed by its first N rows. Hence, the following relationship holds

$$\boldsymbol{A}^H(1{:}\ N, :\ )\ \boldsymbol{w}_{SSB} = \begin{bmatrix}\boldsymbol{a}_d^H(1{:}\ N, :\ )\boldsymbol{w}_{SSB}\\ \boldsymbol{A}_I^H(1{:}\ N, :\ )\boldsymbol{w}_{SSB}\end{bmatrix} \approx \begin{bmatrix}\alpha_1^*\boldsymbol{a}_1^H(\theta_1)\boldsymbol{w}_{SSB}\\ \boldsymbol{0}_{Q\times 1}\end{bmatrix} = \begin{bmatrix}\alpha_1^*\\ \boldsymbol{0}_{Q\times 1}\end{bmatrix} \tag{10}$$

Denoting $\boldsymbol{w}_{ini} = [\boldsymbol{w}_{SSB}^T \ \boldsymbol{0}_{1\times(M-N)}]^T$, then we have

$$\begin{aligned}\boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H\boldsymbol{w}_{ini} &= \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H(1{:}\ N, :\ )\ \boldsymbol{w}_{SSB}\\ &= \begin{bmatrix}\boldsymbol{a}_d & \boldsymbol{A}_I\end{bmatrix}\begin{bmatrix}\sigma_d^2 & \boldsymbol{0}_{1\times Q}\\ \boldsymbol{0}_{Q\times 1} & \boldsymbol{R}_I\end{bmatrix}\begin{bmatrix}\alpha_1^*\\ \boldsymbol{0}_{Q\times 1}\end{bmatrix}\\ &= \sigma_d^2\alpha_1^*\boldsymbol{a}_d\end{aligned} \tag{11}$$

Based on (11), it is evident $\boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H\boldsymbol{w}_{ini}$ is a scaled version of the ideal steering vector $\boldsymbol{a}_d$, with which we can construct an optimal