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Analysis of the dynamics of a memoryless nonlinear gradient IIR adaptive notch filter

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ABSTRACT

A nonlinear dynamical model of a memoryless nonlinear gradient IIR adaptive notch filter for estimating the frequency of a noisy sinusoid is derived. The model is verified through simulations, where simulated responses of the estimated frequency are compared with the responses obtained from the model with good agreement. Convergence properties of the filter are studied using the model, and maximum step sizes and initial frequency ranges for convergence are determined. The performance of the adaptive filter in tracking a time-varying signal frequency is also examined.

1. Introduction

Adaptive estimation of the frequency of a single-tone sinusoid has applications in many fields, such as radar, sonar, communications and biomedical engineering. The infinite-impulse-response (IIR) notch filter is preferred for this task, as it provides an important advantage over its FIR counterpart. This advantage is that the number of parameters required for adaptation is much less than that of the FIR filter, while similar performance is achievable.

Several adaptive algorithms have been developed for the IIR notch filter, such as the sign algorithm (SA) [1,2], the plain gradient (PG) algorithm [3,4], the normalized gradient (NG) algorithm [5], the recursive prediction error (RPE) [6] and other approaches [7,8]. Analyses of these algorithms have been performed, which mainly involve derivation of the frequency estimation bias and variance [3,4,9]. On the other hand, very limited work has been done on the analysis of the large-signal dynamics of adaptive algorithms developed so far. Cho and Lee [10] perform the tracking analysis of the adaptive lattice notch filter for linear chirp signals (linear variation in signal frequency) and for random variation in frequency. Xiao et al. [11] analyze the tracking performance of the IIR adaptive notch filter (IIR-ANF) with constrained poles and zeros, again for the case of linear frequency variation. The analysis is restricted to the asymptotic response and aims mainly at obtaining the tracking bias and the mean-square tracking error in the steady state. Regalia [12] describes the convergence analysis of a lattice-based IIR adaptive notch filter based on continuous-time differential equation approximation of the update equation. Werter [13] uses a similar approach in analyzing the convergence of a digital filter for frequency shift keying (FSK) demodulation.

In this paper, a dynamical model for the constrained IIR notch filter of the memoryless nonlinear gradient (MNG) type [14] is derived. The MNG adaptation algorithm has been shown to have better performance than most of the other algorithms (SA, PG, NG, etc.) at the expense of a slightly increased computational cost [14]. In this work, the derivation of the dynamical model is based on decomposing the error signal into deterministic and random components and representing the deterministic part by a sinusoid with time-varying amplitude. Subsequently, time averaging is applied to obtain a time-invariant nonlinear model of the adaptive filter. The model is complicated and

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does not lend itself to analytical treatment. Therefore, dynamical properties of the filter are studied using graphical methods.

The paper is organized as follows: In Section 2, an overview of the memoryless nonlinear gradient algorithm is given. Analysis of the algorithm takes place in Section 3, where the dynamical model of the adaptive filter is obtained and verified through numerical simulations. Convergence analysis of the filter is also discussed in Section 4 and a general criterion is given for global stability. In Section 5, the adaptive filter's response to sinusoidal variation in signal frequency is investigated. Finally, some conclusions are drawn in Section 6.

2. Problem formulation

Consider a second order adaptive notch filter of the IIR type with constrained poles and zeros, which has the following transfer function from the input signal $x(n)$ to its output $e(n)$:

$$H(z)=\frac{E(z)}{X(z)}=\frac{1+\beta z^{-1}+z^{-2}}{1+r\beta z^{-1}+r^2z^{-2}} \tag{1}$$

where β is the parameter to be adapted and r is the pole contraction factor, which determines the bandwidth of the filter. At a time instant n the error is computed as

$$e(n)=x(n)+x(n-2)+\beta(n)s(n)-r^2e(n-2) \tag{2}$$

where $s(n)$ is defined as the gradient signal and is given by

$$s(n)=x(n-1)-re(n-1). \tag{3}$$

The memoryless nonlinear gradient algorithm for the adaptation of the frequency parameter is given by

$$\beta(n+1)=\beta(n)-\mu\frac{e(n)s(n)}{\varepsilon+s^2(n)} \tag{4}$$

where ε is a small positive number, and $\beta(n)=-2\cos(\omega(n))$, where $\omega(n)$ is the frequency estimate at time instant n.

3. Derivation of the dynamical model

Derivation of the model is performed in three steps. In the first step, the error signal is expressed in terms of deterministic sinusoidal components with time-varying amplitudes, and a noise component. The filter equations are then rewritten in terms of these components. In the second step, the statistical expectation of the update Eq. (4) is taken at any time instant n, using an approximate power series expansion for the reciprocal function. The resulting equations are then averaged in time in the third step.

3.1. Decomposition of the error signal

Let us assume that the input to the adaptive filter is a single sinusoid corrupted by white noise

$$x(n)=A\cos(n\omega_0+\varphi)+w(n) \tag{5}$$

where A is the amplitude, ω_0 is the frequency, φ is the phase and $w(n)$ is a white noise sequence, assumed to be Gaussian. The signal-to-noise ratio (SNR) of the signal in Eq. (5) is defined as

$$\mathrm{SNR}=10\log_{10}\left(\frac{A^2}{2\sigma_w^2}\right)\,dB \tag{6}$$

where σ_w^2 is the variance of the noise $w(n)$. Let the error signal be expressed in the form

$$e(n)=\alpha_c(n)\cos n\omega_0+\alpha_s(n)\sin n\omega_0+v(n) \tag{7}$$

where $v(n)$ is the noise component. Substituting Eqs. (5) and (7) in Eq. (2), we get

$$\begin{aligned}\alpha_c(n)\cos n\omega_0+\alpha_s(n)\sin n\omega_0+v(n)\\=\left\{\begin{array}{l}A(\cos\varphi+\cos(\varphi-2\omega_0)+\beta(n)\cos(\varphi-\omega_0))\\-r\beta(n)(\alpha_c(n-1)\cos\omega_0-\alpha_s(n-1)\sin\omega_0)\\-r^2(\alpha_c(n-2)\cos2\omega_0-\alpha_s(n-2)\sin2\omega_0)\end{array}\right\}\cos n\omega_0\\-\left\{\begin{array}{l}A(\sin\varphi+\sin(\varphi-2\omega_0)+\beta(n)\sin(\varphi-\omega_0))\\+r\beta(n)(\alpha_c(n-1)\sin\omega_0+\alpha_s(n-1)\cos\omega_0)\\-r^2(\alpha_c(n-2)\sin2\omega_0+\alpha_s(n-2)\cos2\omega_0)\end{array}\right\}\sin n\omega_0\\+w(n)+\beta(n)w(n-1)+w(n-2)-r\beta(n)v(n-1)-r^2v(n-2)\end{aligned} \tag{8}$$

Eq. (8) can be decomposed into three components by equating the coefficients of the sinusoidal functions of time, and also equating the random terms on both sides. Let the normalized complex amplitude of the error be defined as

$$\boldsymbol{\alpha}(n)=e^{j\varphi}\frac{\alpha_c(n)+j\alpha_s(n)}{A}=\alpha_1(n)+j\alpha_2(n). \tag{9}$$

The following equations are obtained from Eq. (8) for the complex amplitude and the noise component of the error

$$\boldsymbol{\alpha}(n)+\beta(n)re^{j\omega_0}\boldsymbol{\alpha}(n-1)+r^2e^{j2\omega_0}\boldsymbol{\alpha}(n-2)=1+\beta(n)e^{j\omega_0}+e^{j2\omega_0} \tag{10}$$

$$v(n)+\beta(n)rv(n-1)+r^2v(n-2)=w(n)+\beta(n)w(n-1)+w(n-2) \tag{11}$$

Eqs. (10) and (11) represent an equivalent description of the filter equation in terms of the newly introduced variables, the complex amplitude $\boldsymbol{\alpha}(n)$ and the noise component $v(n)$ of the error signal. Note here that the complex amplitude itself is a random variable due to the random nature of the adaptation process. These equations are not yet in a form suitable for analysis, as they involve random quantities necessitating the application of statistical averaging. Analysis of the adaptive filter would be simplified to a large extent if it is assumed that $\beta(n)$ and the error signal are uncorrelated. This assumption is approximately valid if the step size is sufficiently small, thus leading to slow adaptation of the frequency parameter. Based on this assumption, the expectation of Eq. (10) gives

$$\boldsymbol{\alpha}(n)+\overline{\beta}(n)re^{j\omega_0}\boldsymbol{\alpha}(n-1)+r^2e^{j2\omega_0}\boldsymbol{\alpha}(n-2)=1+\overline{\beta}(n)e^{j\omega_0}+e^{j2\omega_0} \tag{12}$$

where the notation for the complex amplitude is preserved for simplicity, and $\overline{\beta}(n)=E\{\beta(n)\}$. The same assumption makes it possible to derive the statistical parameters of the noise component from Eq. (11). The correlations of the