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S-transform based on analytic discrete cosine transform for time–frequency analysis

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ABSTRACT

This paper presents a new S-transform based on Analytic Discrete Cosine Transform (ADCT). This has been achieved by the use of a Discrete Fourier Transform (DFT) derived from DCT, the analytic DCT which preserves the desirable properties of DCT, viz., the improved frequency resolution and significant reduction in leakage compared to those of conventional DFT. The new S-transform will provide improved performance in terms of frequency resolution and energy concentration (along the frequency axis) as compared to those of S-transform based on DFT. The application of the proposed method to different types of test signals reveals its improved performance in terms of time resolution at high frequencies, frequency resolution in the low frequency region and improved energy concentration.

1. Introduction

The S-transform has gained a major interest in time–frequency representation due to its special features over the conventional wavelet transforms [1]. It provides not only a frequency dependent resolution but also has a direct relation to Fourier spectrum as it is associated with signal absolute phase and frequency indices. However it suffers from poor time resolution at lower frequencies and poor frequency resolution at higher frequencies. These have been improved to certain extent by the use of a customized Gaussian window to provide better energy concentration [2].

For non-stationary signals such as speech, phonocardiogram and electrocardiogram, their statistical properties change with time and hence the time averaged amplitude spectrum obtained using the Fourier transform is inadequate to track the changes in the magnitude spectrum with time. The short time Fourier transform (STFT) often used for non-stationary signal analysis cannot provide simultaneously good frequency resolution and time resolution for low and high frequencies, respectively and this is due to its fixed width window. The wavelet transform (WT) simultaneously provides good frequency and time resolutions for low and high frequencies, respectively as it is equivalent to a STFT using a frequency dependent window. But the conventional continuous wavelet transform (CWT) only provides locally referenced phase information from which the phase referenced to the time origin cannot be recovered. The S-transform (ST) is a time–frequency representation that combines the good characteristics of both STFT and WT. The ST provides frequency dependent resolution maintaining a direct relationship with the Fourier spectrum. Further, the ST can provide the time origin referenced phase information due to the incorporation of Fourier kernel in contrast to locally referenced phase information of CWT [2]. For a specific time instant, the spectrum provided by ST is a one

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dimensional function of frequency and is similar to Fourier spectrum. Hence the ST has been found to be convenient for intuitive visual analysis of bio-signals like phonocardiogram compared to the time-scale plot of WT. The ST has been successfully used to analyze signals in numerous applications, such as cardiovascular time series [3], pattern recognition, geophysics [4], functional magnetic resonance imaging (MRI) [5], complex valued time series and power system disturbance recognition [6]. Sejdic et al. [7] have shown that for the time–frequency representation of heart sound signal, the ST provided the best results compared to those by STFT and WT.

The ST is realized by applying a Gaussian window to the DFT spectrum at each of its frequency bin. However due to the nature of Gaussian window, the ST suffers from poor frequency resolution and energy concentration both in the time and frequency domain. Therefore, different types of windows have been used to improve the time–frequency resolution. McFadden et al. [8] proposed a generalized S-transform which uses a non-symmetric window for resolution control. Two non-symmetric half Gaussian windows have been used by Pinnegar and Mansinha [9]. Some of these methods provide either time or frequency resolution only. Generally in many applications, better resolution in both time and frequency is preferred. In this direction, a new method which controls both time and frequency resolution by a single parameter has been proposed [10]. Further, a Modified S-transform is introduced [11] which provides better energy concentration in the time frequency plane as compared to the original improved S-transform [10]. All the above techniques are based on DFT which suffers from leakage problem or poor energy concentration along the frequency axis and limited frequency resolution. *As the windows used only mask the different frequency bins of the DFT, they at best can only give the basic frequency resolution of the DFT used.* Further as it is known that the DFT suffers from severe leakage effect, the performance of S-transform will be poor. Unless the frequency resolution and leakage effect of the basic DFT are improved, the ST improvement in these factors cannot be achieved and till now such an effort has not been explored for ST. *Hence there is a necessity to improve the basic frequency resolution and energy concentration along the frequency axis of the DFT used, in deriving the ST.*

The DCT due to its symmetrical extension of the signal not only provides twice the frequency resolution but also improves the energy concentration/leakage (due to absence/reduction of signal discontinuity) along the frequency axis compared to those of DFT. With the desired properties of DCT, DFT has been derived and this is nothing but *analytic DCT* (ADCT) [12]. Hence it is of interest to explore the use of ADCT for improving the performance of the ST.

In this paper, a new S-transform based on Analytic DCT (ADCTST) has been proposed. The ADCT provides twice the basic frequency resolution and improved energy concentration along the frequency axis. The application of the proposed ADCTST to non-stationary signals considered indicate that there is a significant performance improvement in terms of frequency resolution and energy concentration compared to those of the existing ST methods.

2. The S-transform

The S-transform is a combination of the STFT and WT. It uses a scalable window length of WT and the Fourier kernel which provides *the phase information referenced to the time origin*. The drawbacks of STFT and WT have motivated the existence of ST. The STFT suffers from the uniform frequency resolution and inability to detect and resolve low frequencies. On the other hand, the WT lacks the absolute phase with respect to a time origin. The ST is a CWT with phase correction [2]. The ST carries out multi-resolution analysis on a time varying signal as its window width varies inversely with frequency. The local scalable Gaussian window dilates and translates in the frequency domain [2]. This gives better time resolution at high frequency and good frequency resolution at low frequency. The choice of Gaussian window is due to the fact that it is symmetric both in time and frequency as the Fourier transform of a Gaussian window is a Gaussian, and there are no side lobes for a Gaussian function.

2.1. The Standard S-transform

The standard S-transform of a signal $x(t)$ is defined as

$$S(\tau, f, \sigma) = \int_{-\infty}^{\infty} x(t) w(\tau - t, \sigma) e^{-j2\pi f t}\, dt \tag{1}$$

where the window function used is a scalable Gaussian window

$$w(t, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2\sigma^2} \tag{2}$$

and

$$\sigma = \frac{1}{|f|} \tag{3}$$

Using Eqs. (2) and (3) in Eq. (1),

$$S(\tau, f, \sigma) = \int_{-\infty}^{\infty} x(t) \left\{ \frac{|f|}{\sqrt{2\pi}} e^{-(\tau - t)^2 f^2/2} e^{-j2\pi f t} \right\} dt \tag{4}$$

The ST given by Eq. (4) has a time localizing Gaussian window that is translated and the Fourier kernel (different from wavelet kernel) that is stationary which localizes the real and the imaginary components of the spectrum independently. The Fourier kernel selects the frequency being localized. Hence it localizes both the amplitude and phase spectrum. Thus it retains absolute phase of the signal which is not provided by WT. However, the Gaussian window has no parameter to control over its width. McFadden et al. [8] and Pinnegar et al. [9] proposed a new ST with a parameter which provides control over the window function width.

2.2. Improved S-transform

In an attempt to control the Gaussian window width, an additional parameter δ was introduced by Sahu [10] and its role in varying width with frequency is given by

$$\sigma(f) = \frac{\delta}{|f|} \tag{5}$$