Signal Processing 105 (2014) 268–276

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

# On change detection in a Kalman filter based tracking problem

B. Moussakhani [a,*], J.T. Flåm [a], T.A. Ramstad [a], I. Balasingham [b]

[a] Department of Electronics and Telecommunications Norwegian, University of Science & Technology, N-7491 Trondheim, Norway
[b] Intervention Center, Rikshospitalet University Hospital, N-0027 Oslo, Norway

ARTICLE INFO

*Article history:*
Received 13 November 2013
Received in revised form
15 May 2014
Accepted 19 May 2014
Available online 2 June 2014

*Keywords:*
Kalman filter
Tracking
Change detection
Multi-mode movement

ABSTRACT

*Objective:* This work considers detecting an additive abrupt state change in a tracking process. It is assumed that the tracking is done by a Kalman filter and that the abrupt change takes place after the steady-state behavior of the filter is reached.
*Result:* The effect of the additive change on the innovation process is expressed in closed form, and we show that the optimal detection method depends on the available information, contained in the change vector.
*Method:* We take a Bayesian perspective and show that prior knowledge on the nature of the change can be used to significantly improve the detection performance.
*Result:* Specifically, we show that performance of such a detector coincides with that of a matched filter when the variance (uncertainty) of the change tends to zero, and it coincides with that of an energy detector when the variance tends to infinity.
*Conclusion:* Finally we conclude that utilizing the derived closed form improves the detection performance for abrupt changes for Kalman filter based tracking problems. In addition, it is concluded that incorporating prior knowledge can improve the detection performance only if the prior variance is less than a certain amount.


## 1. Introduction

The Kalman filter is a strong candidate for tracking mobile nodes in wireless networks [1]. However, mobile nodes may suddenly change their movement pattern while being tracked. The Kalman filter in its basic form will rarely detect such changes quickly; initially the filter will treat such events as spurious noise, rather than a change in the state. The present paper focuses on the detection of impulsive state changes.

Kalman filtering in the presence of changes to the plant and observation equations has been studied during the past decades [2,3] and still attracts interests in new areas [4,5]. The source of the change can be a multitude of causes that distorts the expected evolution, and it can therefore be modeled in different ways (e.g. in state evolution matrix or an additive change vector). In order to capture such changes, different solutions are suggested. In [3], and the references therein, the focus is confined to various *hybrid* estimation approaches. Briefly, hybrid estimation is the estimation of the state vector that has both continuous and discrete components. A wide variety of solutions have been proposed for these types of problems. For example, an adaptive neural controller is suggested in [6] for a nonlinear tracking system and in [7] authors proposed a fuzzy inference system to distinguish between noisy and noise-less pixels in order to remove artifact from images. However, in this work, we confine the study to classical detectors. Which means that, it is first decided if there has been a change in the model and then the state is estimated accordingly. The literature on change detection is rich, and selected works include [2,8].

In the present paper, and in order to detect sudden state changes, we choose to modify the standard plant

* Corresponding author. Tel.: +47 94487074.
*E-mail address:* babak@iet.ntnu.no (B. Moussakhani).

http://dx.doi.org/10.1016/j.sigpro.2014.05.028
0165-1684/

equation. Specifically, we add an extra term which can account for the change. This modification makes it possible, for example, to recognize the movement of a node which stops and starts abruptly. Such changing movement patterns are basic in many widely used movement models, such as random way point (RWP) [9]. The most common method for change detection in the Kalman-based tracking problem is to monitor the energy of the innovation. Once it goes beyond a certain level, one decides that a change has happened. The analysis in [2] shows that by incorporating *prior knowledge* about the nature of the change, the detection performance can be improved significantly.

The present paper studies detection of additive change. The main contributions are as follows. Firstly, we give a closed form expression for the *change signature*. That is, the extra term added to the innovation whenever there is a change. We also present the convergence properties and the impulse response of the change signature. Secondly, assuming *impulsive* change, where a single realization of a Gaussian random vector is added to the plant equation, we present a closed form approximation of the distribution of the Neyman–Pearson detector. In the asymptotic case, when the Gaussian random vector has infinite variance, we show that this detector has no benefit from knowing that the change is impulsive. Instead, it is better to invoke a Generalized Likelihood Ratio Test (GLRT), which neglects the prior information.

The rest of the paper is organized as follows. Section 2 introduces the system model, recalls the standard Kalman filter equations and derives a closed form expression for the change signature. In Section 3 the detection problem is investigated under different levels of prior knowledge about the change. In Sections 4 and 5, some numerical examples are studied, and Section 6 concludes the paper.

## 2. System model

We assume a linear time invariant (LTI) stochastic state-space model where the state and the observation evolve according to the following equations:

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{B}\mathbf{v}_k \tag{1}$$

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{w}_k. \tag{2}$$

Here, $\mathbf{x}_{k\{m\times 1\}}$, $\mathbf{z}_{k\{p\times 1\}}$ are the state and the observation vectors respectively, at time instant $k$. The vectors $\mathbf{v}_{k\{m'\times 1\}}$ and $\mathbf{w}_{k\{p\times 1\}}$ are the process and observation noise respectively, which are assumed to be mutually independent and white, with covariance matrix $\mathbf{Q}$ and $\mathbf{R}$ respectively. The matrices $\mathbf{F}_{\{m\times m\}}$ and $\mathbf{H}_{\{p\times m\}}$ are the state transition matrix and the observation matrix. The matrix $\mathbf{B}_{\{m\times m'\}}$ defines how the process noise influences the state vector. In conformance with much of the literature, we will use upper case boldface letters to denote matrices, and lower case boldface letters to denote column vectors throughout this paper. Lower case italic letters will denote scalars.

We assumed that a Kalman filter is used to track the state vector. We recall that the one-step prediction (3), the innovation (4) and the estimated state (5) can be written as follows:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1}, \tag{3}$$

$$\mathbf{e}_k = \mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}, \tag{4}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{e}_k. \tag{5}$$

Here, the matrix $\mathbf{K}_k$ is commonly called the *Kalman gain* at time instant $k$. The state prediction error covariance matrix $\mathbf{P}_{k|k-1}$, the Kalman gain, and the covariance matrix of the innovation $\mathbf{S}_k$ are given by

$$\begin{aligned}
\mathbf{P}_{k|k-1} &= \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^T + \mathbf{B}\mathbf{Q}\mathbf{B}^T,\\
\mathbf{S}_k &= \mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^T + \mathbf{R},\\
\mathbf{K}_k &= \mathbf{P}_{k|k-1}\mathbf{H}^T\mathbf{S}_k^{-1},
\end{aligned} \tag{6}$$

The Kalman filter is the Minimum Mean Square Error (MMSE) state estimator for this tracking problem, provided that the noise sources are Gaussian. If the noise sources are non-Gaussian, but have known first and second moments, the Kalman filter corresponds to the *linear* MMSE estimator.[1] The main focus of this work is the case when an abrupt change is added to the state equation. Therefore we rewrite the state evolution equation as

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{v}_k + \mathbf{d}_{k,j}, \tag{7}$$

where $\mathbf{d}_{k,j}$ is a vector called the *dynamic profile of the change* at time instant $k$ while the change starts at time instant $j$. This paper considers the problem of detecting the abrupt additive change in a state space setting under different levels of the available prior knowledge about $\mathbf{d}$.

### 2.1. Change signature

Since any deviation from the plant equation in (1) manifests itself by changing the statistics of the innovation process, change detection methods take $\mathbf{e}$ as the observation vector. Therefore finding the relationship between $\mathbf{e}$ and the dynamic profile $\mathbf{d}$ is of pivotal interest. To that end, we assume that (i) the Kalman filter operates in a steady state condition where $\mathbf{K}_k = \mathbf{K}_\infty = \mathbf{K}$, and (ii) the additive change starts at time $j$ and the plant equation follows (7). Under these assumptions it is straightforward to verify that the state evolves after the change as

$$\begin{aligned}
\mathbf{x}_k &= \mathbf{x}_k^0 + \mathbf{a}_{k,j}\\
\mathbf{a}_{k,j} &= \sum_{i=1}^{k-j} \mathbf{F}^{i-1}\mathbf{d}_{k-i,j},
\end{aligned} \tag{8}$$

here, the quantities with the exponent 0 correspond to the system without the change where the plant equation still follows (1). Thus in (8), $\mathbf{x}_k^0$ represents the state vector at time instant $k$ if there was no additive change at time instant $j$. We now claim that the estimated state will have the following form:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k}^0 + \mathbf{b}_{k,j}. \tag{9}$$

[1] Among all estimators which are linear (affine) in the observations, the *linear* MMSE estimator obtains the smallest MSE.