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Distributed greedy pursuit algorithms

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ABSTRACT

For compressed sensing over arbitrarily connected networks, we consider the problem of estimating underlying sparse signals in a distributed manner. We introduce a new signal model that helps to describe inter-signal correlation among connected nodes. Based on this signal model along with a brief survey of existing greedy algorithms, we develop distributed greedy algorithms with low communication overhead. Incorporating appropriate modifications, we design two new distributed algorithms where the local algorithms are based on appropriately modified existing orthogonal matching pursuit and subspace pursuit. Further, by combining advantages of these two local algorithms, we design a new greedy algorithm that is well suited for a distributed scenario. By extensive simulations we demonstrate that the new algorithms in a sparsely connected network provide good performance, close to the performance of a centralized greedy solution.

1. Introduction

Compressed sensing (CS) [1,2] refers to an under-sampling problem, where few samples of an inherently sparse signal are collected via a linear measurement matrix with the objective of reconstructing the full sparse signal from these few samples. Considering the fact that sparsity is ubiquitous in nature, CS has many potential applications. In the literature, the task of developing CS reconstruction algorithms has presumably been considered for a set-up where the samples are acquired by using a single sensor. In the CS community, we note that there is an increasing effort to consider a multiple-sensor setup.

For a multiple-sensor setup, an interesting case is a distributed setup where several CS-based sensors are connected through a distributed (decentralized) network. Such a setup is useful in a wide range of applications, for example in distributed sensor perception [3] and distributed spectrum estimation [4–6]. Considering a camera sensor network, we can envisage a scheme where a set of measurement samples (i.e., CS samples of image signals) from different angles at different positions are acquired. Instead of reconstructing the underlying signals from the corresponding samples independently, one could potentially improve the quality of the reconstructed signals by taking into account all the measurement samples. This is possible by exchanging information over the (connected) network. We refer to this problem as distributed CS (DCS) problem, where the sensor nodes are connected with an arbitrary network topology. In the DCS problem, the sparse signals acquired at the sensors are correlated. If all sensors transmit their measured samples to a common centralized point, the problem can be solved by a centralized algorithm. For such a setup, we have recently developed joint greedy pursuit reconstruction algorithms in [7]. In the literature, we find several other attempts for centralized solutions with various model assumptions [8,9]. Additionally the works based on simultaneous sparse approximation (SSA) [10,11] and multiple measurement vector (MMV) [12,13] problems, for example simultaneous

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orthogonal matching pursuit (SOMP) algorithm [14], can be considered to be applied for a centralized (or joint) CS setup. The paper [15] provides a good overview comparing several centralized algorithms.

When a centralized setup is not possible for the DCS problem, a distributed CS setup may be a good choice. For the distributed CS setup, we notice some recent attempts to design convex relaxation algorithms [4–6,16]. A non-convex algorithmic approach which attempts to minimize an ℓ_q minimization problem distributively is presented in [17]. While the convex relaxation algorithms are theoretically elegant and provide good practical performance for low dimensional problems, their use for high dimensional problems are limited due to their high complexity (here, a high dimensional problem refers to the case where the dimensions of underlying signals are high). Typically the complexity of a convex relaxation algorithm scales with signal dimension N cubically as $\mathcal{O}(N^3)$ [18]. Naturally, designing computationally simple greedy pursuit (GP) (also called greedy search) algorithms is an attractive alternative. In general, a GP algorithm uses computationally simple detection and estimation techniques iteratively and hence they are computationally efficient for higher dimensional problems. Typically the complexity for standard GP algorithms is $\mathcal{O}(N \log N)$ [19]. While there exist several centralized GP algorithms for the DCS problem, such as [7,14,20,13], there is so far few attempts for solving the DCS problem based on distributed GP algorithms. We first addressed this problem in [21] and we found another recent contribution in [22].

In this paper, we develop GP algorithms for solving the DCS problem where each node reconstructs a signal which is correlated with signals stemming from other sensor nodes. We refer to the new algorithms as distributed GP (DiGP) since there is no centralized node. For the correlations in the DCS problem considered here, we first introduce a mixed support-set signal model [7], where the correlations are modeled as overlap in the support-set of the signals at different nodes. We claim that this new signal model is less restrictive compared to previous signal models [9,12,23] in the literature. Based on this signal model, we develop three DiGP algorithms. Two of the DiGP algorithms are built upon existing GP algorithms by introducing appropriate modifications. The existing GP algorithms which we modify are orthogonal matching pursuit (OMP) [24] and subspace pursuit (SP) [25]. We also develop a new GP algorithm which we call FROGS, by combining strengths from both OMP and SP, and then use FROGS to develop the third DiGP algorithm. Based on surveys on DCS convex and GP algorithms, and by introduction of a new signal model, the contributions of this paper are:

- Development of the new GP algorithm FROGS.
- Development of three new distributed greedy pursuit algorithms.

We provide several results through simulations: average number of algorithm iterations, algorithm execution times, algorithm performance over iterations, algorithm performance over several kinds of networks. Inspiration for the work in this paper came since the authors were working with improving the performance of GP algorithm for standard CS (i.e., [19]) and from work with the centralized joint sparse signal recovery [7]. Importantly we mention that correlation structure in the new signal model is realized via a generalization of existing models used in sparse signal recovery and jointly sparse signal recovery schemes. In the new signal model used for DCS, all signals observed at all sensors have a common sparsity pattern along with each signal at each node has its own private sparsity pattern.

The remaining parts of the paper are arranged as follows: In Section 1.1, we present the general DCS problem and in Sections 1.2 and 1.3 we provide literature surveys for convex and greedy algorithms, respectively. We introduce the new signal model for distributed CS in Section 2; the section also includes a structured approach for describing the quality of connectivity in a distributed network. In Section 3, we introduce the new stand-alone GP algorithm. In Section 4, we introduce the concept of DiGP and develop the three DiGP algorithms. In Section 5, we evaluate the average number of iterations for the proposed algorithms. We end the paper with experimental evaluations in Section 6.

Notations: Let a matrix be denoted by a upper-case bold-face letter (i.e., $\mathbf{A} \in \mathbb{R}^{M \times N}$) and a vector by a lower-case bold-face letter (i.e., $\mathbf{x} \in \mathbb{R}^{N \times 1}$). $\mathcal{T}$ is the support-set of $\mathbf{x}$, which is defined in the next section. We also denote $\overline{\mathcal{T}} = \{1, 2, \ldots, N\} \setminus \mathcal{T}$ as the complement to $\mathcal{T}$ where $\setminus$ is the set-minus operator. $\mathbf{A}_{\mathcal{T}}$ is the sub-matrix consisting of the columns in $\mathbf{A}$ corresponding to the elements in the set $\mathcal{T}$. Similarly $\mathbf{x}_{\mathcal{T}}$ is a vector formed by the components of $\mathbf{x}$ that are indexed by $\mathcal{T}$. We let $(\cdot)^\dagger$ and $(\cdot)^T$ denote pseudo-inverse and transpose of a matrix, respectively. We use $\| \cdot \|$ to denote the l_2 norm of a vector.

1.1. Distributed compressed sensing problem

For the distributed CS (DCS) problem, considering the l-th sensor, the sparse signal $\mathbf{x}_l \in \mathbb{R}^N$ is measured as

$$\mathbf{y}_l = \mathbf{A}_l \mathbf{x}_l + \mathbf{w}_l, \quad \forall l \in \{1, 2, \ldots, L\}, \tag{1}$$

where $\mathbf{y}_l \in \mathbb{R}^M$ is a measurement vector, $\mathbf{A}_l \in \mathbb{R}^{M \times N}$ is a measurement matrix, and $\mathbf{w}_l \in \mathbb{R}^M$ is the measurement error. In this setup $M < N$ and hence the system is under-determined. Typically $\mathbf{A}_l$ and $\mathbf{w}_l$ are independent across l. The signal vector $\mathbf{x}_l = [x_l(1)\ x_l(2), \ldots]$ has K_l non-zero components with a set of indices $\mathcal{T}_l = \{i : x_l(i) \neq 0\}$. $\mathcal{T}_l$ is referred to as the support-set of $\mathbf{x}_l$ with cardinality $|\mathcal{T}_l| = K_l$. The DCS reconstruction problem strives to reconstruct $\mathbf{x}_l$ for all l by exploiting some shared structure (correlation) defined by the underlying signal model and by exchanging some information between the nodes (sensors).

Based on (1), we will now provide a literature survey. In this literature survey, we endeavor to distinguish between a distributed and centralized algorithm and also between the distributed and centralized CS problem. A solution algorithm can be either distributed or centralized independent of whether the underlying signals to be estimated are correlated or not. For example, the standard, one-sensor, CS problem can be solved by a distributed