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# A low-complexity variable forgetting factor constant modulus RLS algorithm for blind adaptive beamforming ☆

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ABSTRACT

In this paper, a recursive least squares (RLS) based blind adaptive beamforming algorithm that features a new variable forgetting factor (VFF) mechanism is presented. The beamformer is designed according to the constrained constant modulus (CCM) criterion, and the proposed adaptive algorithm operates in the generalized sidelobe canceler (GSC) structure. A detailed study of its operating properties is carried out, including a convexity analysis and a mean squared error (MSE) analysis of its steady-state behavior. The results of numerical experiments demonstrate that the proposed VFF mechanism achieves a superior learning and tracking performance compared to other VFF mechanisms.


## 1. Introduction

Numerous adaptive beamforming algorithms for application to wireless communication receivers have been reported in the literature in the last few decades [1–4]. In this application, blind adaptation is highly desirable for the digital receivers equipped with an antenna array, since it operates without the training sequences and leads to a good solution. The constrained constant modulus (CCM) criterion [5] is often considered as one of the most promising design criterions for blind beamforming. It takes advantage of the constant modulus (CM) property of the source modulation, while subject to a constraint on the array response to the desired user [3,6]. The work in [7] investigates the CCM-RLS algorithms, which combine the use of RLS adaptation with the CCM criterion, for different applications and shows that the CCM based algorithms generally outperform the ones based on constrained minimum variance (CMV).

The superior performance of the RLS-based blind beamformers is often demonstrated under the assumption of stationarity, where an ideal choice of the forgetting factor can be made. However in reality, it is difficult or even impractical to compute a predetermined value for the forgetting factor [8]. Hence, the use of a variable forgetting factor (VFF) mechanism is an attractive choice to overcome this shortcoming. Among such mechanisms, the most common one proposed in [6] is the gradient-based variable forgetting factor (GVFF), which is varied according to the measured square error at the beamformer output. Recently,

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the authors in [8] have extended the conventional GVFF scheme to the CCM-RLS blind beamformer for direct sequence code division multiple access (DS-CDMA) receiver. In particular, they have proposed a new VFF mechanism that leads to a superior performance yet with a reduced complexity.

The problem formulation using the CCM criterion can be broken down into constrained and unconstrained components that give rise to the generalized sidelobe canceler (GSC) [6] structure. The latter uses a main branch consisting of a fixed beamformer steered towards the desired user, in parallel with adaptive auxiliary branches that have the ability to block the signal of interest; they produce an output which ideally consists only of interference and is subtracted from that of the main branch. To the best of our knowledge, the study of effective VFF mechanisms for CCM-RLS beamformers developed around the GSC structure has not been addressed in the technical literature.

In this work, we present an extension of the method reported in [8] for the direct-form beamformer (DFB) structure to the more practical GSC structure. The difference between these two structures has a major influence on the derivations and expressions of the adaptive weight vectors. In the GSC context, the proposed time-averaged variable forgetting factor (TAVFF) mechanism employs the time average of the CM cost function to automatically adjust the forgetting factor. Then convexity analysis and convergence analysis of the resulting CCM-RLS algorithm with TAVFF are carried out and expressions to predict the steady-state mean squared error (MSE) are obtained. Simulation results are presented to show that the proposed TAVFF mechanism leads to a superior performance of the CCM-RLS beamformer in the GSC structure.

## 2. System model and GSC beamformer design

We consider a wireless communication scenario in which $K$ narrowband user signals impinge on a uniform linear array (ULA) comprised $M$ identical omnidirectional antennas. Let $\lambda_c$ denote the wavelength and $d_s = \lambda_c/2$ be the inter-element spacing of the ULA. Assuming that the $k$th user signal impinges on the array with direction of arrival $\theta_k$, we can write the normalized corresponding steering vector $\mathbf{a}(\theta_k) = (1/\sqrt{M})[1, e^{-j2\pi(d_s/\lambda_c)\cos\theta_k}, \ldots, e^{-j2\pi(d_s/\lambda_c)\cos\theta_k(M-1)}]^T$. Then the sampled array output vector (or snapshot) at discrete time $i \in \mathbb{N}$ can be modeled as

$$\mathbf{r}(i) = \mathbf{A}(\theta)\mathbf{b}(i) + \mathbf{n}(i), \quad i = 0, 1, 2, \ldots \tag{1}$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_0), \ldots, \mathbf{a}(\theta_{K-1})]$ is the matrix of steering vectors, $\mathbf{b}(i) = [b_0(i), \ldots, b_{K-1}(i)]^T$ is the data vector and $\mathbf{n}(i)$ is an additive vector of sensor noise with zero-mean and covariance matrix $\sigma^2\mathbf{I}$, where $\sigma^2$ denotes the variance and $\mathbf{I}$ is an identity matrix of order $M$. We assume that the sequences of transmitted symbols by the desired and interference users are independent and identically distributed (i.i.d.) random processes, with values taken from a constant modulus modulation format.

The GSC structure, illustrated in Fig. 1, converts the constrained optimization problem into an unconstrained one [6]. Its output is given by $y(i) = (v\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H\mathbf{r}(i)$,

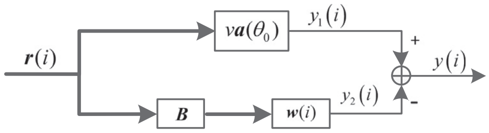


**Fig. 1.** GSC structure for blind beamforming.

where $v$ is a real scalar, the signal blocking matrix $\mathbf{B}$ is orthogonal to the steering vector $\mathbf{a}(\theta_0)$ and $\mathbf{w}(i)$ is the complex adaptive weight vector. In this work, $\mathbf{w}(i)$ is optimized in an adaptive manner according to the CM cost function

$$J_{CM}(\mathbf{w}(i)) = \mathbb{E}[(|y(i)|^2 - 1)^2]. \tag{2}$$

The CCM design has its convexity enforced by adjusting the parameter $v$, as will be discussed along with the analysis in Section 5. The objective of the design based on the CM cost function (2) is to minimize the expected deviation of the square modulus of the beamformer output from a constant while maintaining the contribution from $\theta_0$ constant, i.e., $(v\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H\mathbf{a}(\theta_0) = v$.

## 3. Blind adaptive CCM-RLS-GSC algorithm

For the GSC structure depicted in Fig. 1, by employing the time-averaged estimation, we obtain the following CM cost function:

$$J_{CM}(\mathbf{w}(i)) = \sum_{n=1}^{i} \lambda^{i-n}(|(v\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H\mathbf{r}(n)|^2 - 1)^2, \tag{3}$$

where the forgetting factor $\lambda$ should be chosen as a positive constant. By taking the gradient of (3) with respect to $\mathbf{w}^*(i)$ and equating it to zero, we have

$$\frac{\partial J_{CM}(\mathbf{w}(i))}{\partial \mathbf{w}^*} = \sum_{n=1}^{i} \lambda^{i-n}\left(\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(i) - \mathbf{x}(n)d^*(n)\right) = \mathbf{0}, \tag{4}$$

where $\mathbf{x}(n) = \mathbf{B}^H\tilde{\mathbf{r}}(n)$, $\tilde{\mathbf{r}}(n) = y^*(n)\mathbf{r}(n)$ and $d(n) = v\mathbf{a}^H(\theta_0)\tilde{\mathbf{r}}(n) - 1$. Defining the correlation matrix $\mathbf{Q}(i) = \sum_{n=1}^{i}\lambda(n)^{i-n}\mathbf{x}(n)\mathbf{x}^H(n)$, and cross-correlation vector $\mathbf{p}(i) = \sum_{n=1}^{i}\lambda\,(n)^{i-n}\mathbf{x}(n)d^*(n)$, it follows from (4) that $\mathbf{w}(i) = \mathbf{Q}^{-1}(i)\mathbf{p}(i)$. This expression for $\mathbf{w}(i)$ has the same form as the well-known weighted least-square solution, and hence we can directly obtain the RLS equations [6]

$$\mathbf{w}(i) = \mathbf{w}(i-1) + \mathbf{k}(i)e^*(i), \tag{5}$$

where $\mathbf{k}(i) = \mathbf{Q}^{-1}(i-1)\mathbf{x}(i)/(\lambda + \mathbf{x}^H(i)\mathbf{Q}^{-1}(i-1)\mathbf{x}(i))$, and $e(i) = d(i) - \mathbf{w}^H(i-1)\mathbf{x}(i)$. These equations with proper initialization define the CCM-RLS blind beamforming algorithm for the GSC structure.

## 4. Proposed TAVFF scheme

### 4.1. Blind TAVFF mechanism

Motivated by the variable step size (VSS) mechanism for the least mean square (LMS) algorithm [6] and the original work in [8], we introduce a new variable quantity