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Edge structure preserving image denoising

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ABSTRACT

Image denoising is important in image analysis. It is often used for pre-processing images so that subsequent image analysis is more reliable. Besides noise removal, one important requirement for image denoising procedures is that they should preserve true image structures, such as edges. This paper proposes a novel denoising procedure which can preserve edges and major edge features (e.g., angles of the edges). Our method is based on nonparametric estimation of a discontinuous surface from noisy data, in the framework of jump regression analysis, because a monochrome image can be regarded as a surface of the image intensity function and such a surface has discontinuities at the outlines of objects. Numerical studies show that this method works well in applications, compared to some existing image denoising procedures.

1. Introduction

Image denoising is often used for pre-processing images so that subsequent image analysis is more reliable [1]. Besides noise removal ability, another important requirement for image denoising procedures is that true image structures, such as edges, should be preserved in the denoising process. In this paper, we handle the image denoising problem in the framework of jump regression analysis (JRA), which is a research area handling regression models involving jumps and discontinuities [2]. In this framework, image denoising can be accomplished by estimating a discontinuous surface from noisy data, because a monochrome image can be regarded as a surface of the image intensity function and such a surface has discontinuities at the outlines of objects. A novel procedure is suggested in this paper for estimating discontinuous surfaces from noisy data, which can preserve edges and major edge features (e.g., angles of the edges).

In the literature, there are some existing procedures for image denoising and restoration. One group of methods are based on Bayesian estimation, using Markov random field (MRF) modeling and maximum *a posteriori* (MAP) algorithms (e.g., [3–9]). Some closely related methods use the regularization approach, by minimizing certain objective function that enforces a roughness penalty in addition to a term measuring fidelity of an estimator to the data (e.g., [10,11]). Image denoising by median filtering and robust estimation is a popular pre-smoothing tool in image processing, because it has certain ability of preserving edges when removing noise (e.g., [12–15]). Other image restoration procedures include adaptive smoothing filters (e.g., [16,17]), bilateral filtering procedures (e.g., [18,19]), diffusion filtering procedures (e.g., [20,21]), wavelet transformation procedures (e.g., [22–25]), discontinuity-preserving surface estimation procedures (e.g., [26–31]), among some others. See Qiu [32] for a more detailed discussion on this topic.

Most image denoising and jump surface estimation procedures mentioned above have ability in preserving edges at places where the edge curvature is not large. At places where the edges have angles or where their curvature is large, however, such edges are often

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blurred or rounded by these existing methods (cf., some numerical examples in Section 3). One major reason why this would happen is that the edge structures (e.g., angles) are hidden in observed image intensities, they are not easy to describe and measure (cf., [33,34]), and they are even more difficult to accommodate in the image denoising process (cf., [26]). In our opinion, edge structures are an important part of images, because they often denote major characteristics of image objects, and are easier to capture our visual attention than the parts of the edges with relatively small curvature. Therefore, they should be preserved during image denoising. In other words, a good image denoising procedure should preserve not only the parts of the edges with small curvature but also certain major edge structures, such as angles, corners, and other places on the edges with large curvature, although the latter goal is much more challenging than the former.

In this paper, an image denoising procedure is suggested, which can preserve edges and major edge structures well. Our method is based on JRA, and consists of three major steps, outlined below. First, edge pixels are detected in the whole design space by an edge detector. Second, in a neighborhood of a given pixel, a piecewise linear curve is estimated from the detected edge pixels by a simple but efficient algorithm, to approximate the underlying edge segment in that neighborhood. Finally, observed image intensities on the same side of the estimated edge segment, as the given pixel, are averaged by the local linear kernel smoothing procedure (cf., [35]), for estimating the true image intensity at the given pixel. This proposed image denoising procedure is described in detail in Section 2. Some numerical examples are presented in Section 3, for evaluating its numerical performance, in comparison with several existing denoising procedures. Some remarks conclude the article in Section 4.

2. Methodology

We present our proposed methodology in three parts. In Section 2.1, 2-D local quadratic kernel (LQK) smoothing and a corresponding edge detection procedure is introduced. Local approximation to edge segments and edge-structure-preserving local denoising are described in Section 2.2. Data driven parameter selection is discussed in Section 2.3.

2.1. Edge detection by LQK smoothing

As discussed in Section 1, the first step of the proposed image denoising procedure is to detect edge pixels using an edge detector. Theoretically speaking, any reasonable edge detector can be used here. In the literature, most existing edge detectors are based on estimation of the first-order derivatives (e.g., [36–39]) or the second-order derivatives (e.g., [40,41]) of the image intensity function. Recently, Sun and Qiu [42] propose an edge detector that combines the major strengths of the two types of edge detectors, by using both the first-order and the second-order derivatives of the image intensity function. This edge detector will be used in all numerical examples of this paper, and it is briefly described below.

Assume that observed image intensities $\{Z_{ij}, i,j=1,2,\ldots,n\}$ follow the following 2-D regression model:

$$Z_{ij}=f(x_i,y_j)+\varepsilon_{ij},\quad \text{for } i,j=1,2,\ldots,n, \tag{1}$$

where $\{(x_i,y_j),\ i,j=1,2,\ldots,n\}$ are equally spaced pixel locations, f is the unknown image intensity function, and $\{\varepsilon_{ij}, i,j=1,2,\ldots,n\}$ are independent and identically distributed (i.i.d.) random errors with mean 0 and unknown variance σ^2. At a given pixel (x,y), let us consider a circular neighborhood $O^*(x,y)=\{(u,v):\sqrt{(u-x)^2+(v-y)^2}\le h_n^*\}$, where $h_n^*>0$ is a bandwidth parameter. Then, LQK smoothing is accomplished by

$$\min_{a,b,c,d,e,f}\sum_{(x_i,y_j)\in O^*(x,y)}\{Z_{ij}-[a+b(x_i-x)+c(y_j-y)+d(x_i-x)(y_j-y)$$
$$+e(x_i-x)^2+f(y_j-y)^2]\}^2K\left(\frac{x_i-x}{h_n^*},\frac{y_j-y}{h_n^*}\right), \tag{2}$$

where K is a radially symmetric, bivariate density kernel function with support $\{(x,y):x^2+y^2\le 1\}$. The solution to a of the minimization problem (2) can be used as an estimator of the intensity $f(x,y)$, the solution to $(b,c)'$ as an estimator of the gradient vector $G(x,y)=(f_x(x,y),f_y(x,y))'$, and the solution to $(e,f)'$ as an estimator of $(f_{xx}(x,y), f_{yy}(x,y))'$. These estimators, denoted as $\hat{f}(x,y), \hat{f_x}(x,y), \hat{f_y}(x,y), \hat{f_{xx}}(x,y)$, and $\hat{f_{yy}}(x,y)$, are called LQK estimators in the literature (e.g., [35]). Along the estimated gradient direction $\hat{G}(x,y)=(\hat{f_x}(x,y),\hat{f_y}(x,y))'$, if (x,y) is on an edge segment, then $\hat{G}(x,y)$ would have large Euclidean length and $(\hat{f_{xx}}(x,y),\hat{f_{yy}}(x,y))'$ would have the *zero-crossing* properties that they are zero at (x,y) and change signs on two different sides of the edge segment. See Fig. 1 for a demonstration in one-dimensional cases. Then, a point (x,y) is flagged as a detected edge pixel if $\sqrt{\hat{f_x}^2(x,y)+\hat{f_y}^2(x,y)}$ is larger than a threshold value u_n and $\hat{f_{xx}}+\hat{f_{yy}}$ demonstrates the zero-crossing properties in $O^*(x,y)$. A formula for the threshold value is derived in Sun and Qiu [42], which depends on a significance level α_n. In all our numerical examples presented in Section 3, α_n is fixed at 0.01.

2.2. Edge structure preserving image denoising

Detected edge pixels are identified after the edge detection step discussed in the previous part. In this part, we describe the remaining two steps of the proposed image denoising procedure. At a given pixel (x,y), we consider its circular neighborhood

$$O(x,y)=\{(u,v):\sqrt{(u-x)^2+(v-y)^2}\le h_n\},$$

where $h_n>0$ is a bandwidth parameter which could be different from h_n^* used in (2). Detected edge pixels in $O(x,y)$ are denoted by $\{(w_k,v_k),\ k=1,2,\ldots,m\}$. Our major goal here is to estimate $f(x,y)$ from observations in $O(x,y)$ with possible edges preserved.