Signal Processing 104 (2014) 136–142

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Ambiguity in range–Doppler determination using waveforms of a solvable chaotic oscillator

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ARTICLE INFO

Article history:
Received 3 December 2013
Received in revised form
21 February 2014
Accepted 22 March 2014
Available online 29 March 2014

Keywords:
Ambiguity function
Chaos
Radar
Nonlinear
Pulse compression
Phase coded waveforms

ABSTRACT

The ambiguity function is derived analytically for waveforms from a chaotic oscillator that has an analytic solution. The chaotic solutions of this oscillator can be expressed as a superposition of basis functions, similar to conventional communication or phase coded radar waveforms. Example waveforms are considered to illustrate the variety of ambiguity functions obtainable from a free running oscillator. The mean and the variance of the ambiguity function for waveforms generated by a free running oscillator are derived to determine typical performance. The mean ambiguity function is shown to have a single, localized peak with low variance indicating that solvable chaos has significant potential as the basis of novel remote sensing technologies.

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1. Introduction

The chaotic oscillations displayed by nonlinear dynamical systems have many of the properties desired in a ranging waveform. These oscillations are broadband and non-repeating, precisely the characteristics required for high-resolution, unambiguous range determination. Chaotic oscillations can be generated by very simple nonlinear devices which may allow for savings in terms of cost and complexity of signal generation. Chaotic oscillators can also be synchronized to allow for power combining and beam forming [1,2]. These properties have motivated a body of exploratory research into ranging using chaos [3–14]. A significant barrier to chaotic ranging is the difficulty of theoretical analysis of oscillators for which no analytical solutions can be found. Generally, chaotic dynamical systems are only susceptible to numerical solution techniques. However, some exceptions to this rule have recently been reported [15,16]. These surprising systems have chaotic solutions that can be expressed as a sum of copies of a single basis function evenly spaced in time with a randomly varying amplitude, much like a conventional communication or radar waveform. The basis function allows the implementation of simple matched filters for optimal reception of these waveforms in the presence of noise. One such *solvable chaotic oscillator* has been used in an experimental demonstration of acoustic ranging [17].

In this paper, we exploit the explicit solution of this oscillator to determine the ambiguity function of its waveform analytically. This theoretical result is important because limits on the resolution and ambiguity of range and Doppler measurements follow from an analysis of the ambiguity function. In addition, we show that statistical properties of the ambiguity function can be determined analytically as well, such as the mean and variance of the main peak. These statistics provide information on the typical resolution and ambiguity of waveforms generated

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http://dx.doi.org/10.1016/j.sigpro.2014.03.032
0165-1684/Published by Elsevier B.V.

by a free running oscillator. Previous researchers have examined ambiguity functions for other chaotic systems, but lacking an analytic solution they were forced to rely on numerical simulations or experimental data [6,10,11]. Thus for chaotic systems, the level of rigor achieved here is unprecedented.

2. Wideband ambiguity function

A wideband signal reflected from a moving target experiences a dilation in time due to the Doppler effect. The range and the radial velocity of the target can be determined from the measured time of flight and dilation of a received echo, respectively. For a wideband signal $u(t)$, the ambiguity function may be defined as [18–20]

$$A(\tau,\gamma)=\frac{1}{\sqrt{\gamma}}\int_{-\infty}^{\infty}u(t)u\left(\frac{t+\tau}{\gamma}\right)dt, \tag{1}$$

where τ is the time of flight and $\gamma=(c+v)/(c-v)$ is the Doppler stretch factor, where c is the wave speed and v is the relative radial velocity of the target. The factor of $\sqrt{\gamma}$ is included to renormalize the signal power of the dilated signal. This formulation assumes that the target velocity is constant over the period of observation. The wideband ambiguity function necessarily takes on its maximum value for $\tau=0$ and $\gamma=1$. The ideal function is thumbtack shaped, with a unique, sharp peak around this maximum [18]. In practice, the main peak of any ambiguity function has some finite width, and other peaks of lesser height are always present. The width of the main peak indicates the degree to which two closely spaced targets with similar ranges or velocities can be resolved. Additional maxima in the ambiguity function define the limits within which unambiguous range and velocity determinations can be made. Strictly speaking, resolution is only limited by the presence of noise. But the ambiguity function gives a practical estimate of the limits of resolution and ambiguity.

As the characteristics of the wideband ambiguity function directly impact system performance, it is desirable to be able to analyze them rigorously. In general, if a chaotic or random source is used to generate the waveform, no explicit analytic representation of the waveform is available. Thus, the ambiguity function must be approximated using simulated or experimentally recorded waveforms [6,10,11]. Statistical properties of the ambiguity function can only be determined using large numbers of sampled waveforms. In the next section, we examine an unusual chaotic system for which an analytic representation of the waveform is, in fact, available. This expression will later allow us to derive an exact expression for the ambiguity function itself, as well as its mean and variance.

3. A solvable chaotic oscillator

Here we describe a chaotic dynamical system that is unusual in that an analytic solution is known. An analytic solution is rare among chaotic systems, but recently a class of solvable hybrid oscillators has been investigated [15,16,21]. The oscillator we focus on here is a hybrid dynamical system defined by a differential equation along with a guard condition as follows. The differential equation is the linear, second order equation

$$\frac{d^2u}{dt^2}-2\beta\frac{du}{dt}+\left(\omega^2+\beta^2\right)\cdot(u-s)=0, \tag{2}$$

where $u(t)$ is a continuous state variable and $s(t)$ is a discrete state. In electronic implementations, $u(t)$ represents a voltage across a capacitor [15,22–25]. Throughout this paper, we fix $\omega=2\pi$ in order to conveniently scale time such that the oscillation period is one time unit. Also, $0<\beta\le\ln 2$ is a parameter whose value will be discussed further below.

The discrete state $s(t)$ is updated when the *guard condition* is satisfied. A variety of topologically distinct oscillations are possible depending on the exact details of the guard condition [15,16,22,23]. The oscillation of interest here results from the condition

$$\frac{du}{dt}=0\Rightarrow s(t)=\mathrm{sgn}(u(t)), \tag{3}$$

meaning that the discrete state $s(t)$ is set to the sign of $u(t)$, i.e. $+1$ or -1, whenever the derivative of $u(t)$ vanishes. A phase projection of a typical chaotic oscillation of this system with $\beta=\ln 2$ is shown in Fig. 1. More generally, the system is known to be chaotic for any $\beta\in(0,\ln 2]$. This system has been realized in a number of electronic circuits, as well as in an electro-mechanical oscillator [15,22–26].

It has been shown that a solution of this oscillator can be expressed in terms of a single basis function as [15,21]

$$u(t)=\sum_{m=-\infty}^{\infty}\sigma_m\cdot P(t-m), \tag{4}$$

where the basis function is

$$P(t)=\begin{cases}P_1(t), & t<-1/2\\ P_2(t), & -1/2\le t<1/2\\ 0, & 1/2\le t\end{cases}, \tag{5}$$

where

$$P_1(t)=-e^{\beta(t+1/2)}\left(1-e^{-\beta}\right)\left(\cos\,\omega t-\frac{\beta}{\omega}\sin\,\omega t\right), \tag{6}$$

and

$$P_2(t)=1+e^{\beta(t-1/2)}\left(\cos\,\omega t-\frac{\beta}{\omega}\sin\,\omega t\right). \tag{7}$$

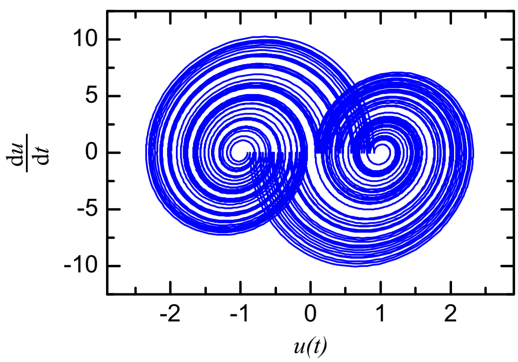

Fig. 1. Phase space projection of a typical chaotic oscillation with $\beta=\ln 2$.