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State filtering and parameter estimation for state space systems with scarce measurements ☆

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ABSTRACT

This paper considers the state filtering and parameter estimation problems for state space systems with scarce output availability. When the scarce states are available, a least squares based algorithm and an observer based parameter estimation algorithm are developed to estimate the system parameter matrices and states. For the case with unknown states, a combined parameter estimation and state filtering algorithm is presented for canonical state space models, using the reconstructed states for the parameter estimation. Finally, an example is provided to test the effectiveness of the proposed algorithms.

1. Introduction

Parameter estimation has wide applications in signal filtering [1–3,4], control theory [5,6], state estimation [7,8] and system identification [9–11]. Many estimation methods have been proposed for linear or nonlinear systems described by the transfer function models or state space models [12,13]. However, most methods assume that all input and output data are available at every instant [14]. This paper considers identification problems for systems with scarce measurements or with missing data.

The reconstruction of the missing data is a basic approach in signal filtering or signal modeling, e.g., [15–17]. The reconstruction methods include the interpolation values between two available measurements by linear, parabola, cubic, spline or hold interpolation. The missing data can be reconstructed with a dynamical model. In this aspect, the expectation maximization (EM) approach can be used for finding maximum likelihood (ML) estimates of the parameters of a state space system and for estimating the missing samples [18].

For systems with scarce measurements, Raghavan et al. studied the EM-based state space model identification problems with irregular output sampling [19]; Sanchis and Albertos discussed the recursive identification under scarce-data operation [20]. Recently, the auxiliary model based least squares method was developed for system identification with irregularly missing data [21]; Ding et al. studied the reconstruction problems of continuous-time systems from their non-uniformly sampled discrete-time systems [22].

The dual-rate sampled systems can be regarded as a class of systems with regular missing data [21,23,24]. The non-uniformly sampled systems can also be viewed as a class of non-uniformly sampled systems or systems with scarce measurements [25]. Recently, a gradient-based algorithm has been developed to estimate the parameters of the input–output representation for systems with scarce

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measurements, and the convergence properties of the parameter estimation and unavailable output estimation are established using the Kronecker lemma and the deterministic version of the martingale convergence theorem [26]. On the basis of the work in [26], this paper studies parameter identification methods for state space systems with irregular missing data patterns or scarce sampling patterns. Different from the ARX models in [17,20], this paper uses the state space output error models with scarce measurements.

The rest of this paper is organized as follows. Section 2 introduces the identification problems to be discussed for systems with scarce measurements. Sections 3 and 4 discuss the least squares based parameter identification and the observer based parameter identification using a state estimator for unavailable states, assuming that the scarce states are available. Section 5 presents a combined state and parameter estimation algorithm, supposing that the states are completely unavailable. Section 6 gives an illustrative examples to show the effectiveness of the proposed algorithms. Finally, we offer some concluding remarks in Section 7.

2. Problem description

In order to distinguish systems with scarce measurements from systems with missing data, we roughly define them [21,26,27]. In general, a system is said to be a missing data system if missing data are only a small part compared with available data, as shown in Fig. 1 [27], where the outputs $y(4)$, $y(9)$, $y(10)$, $y(16)$, $y(22)$, etc. are missed. A system is said to be a system with scarce measurements if most data are missing and a few data are available over a period of time, as shown in Fig. 2.

Different from the missing-data systems in [21], this paper considers such a system with scarce measurements that the input $u(t)$ is available at every instant t because the input signals are usually generated by digital computers in practice and are normally available, and only scarce measurement data are available, e.g., $y(0)$, $y(1)$, $y(3)$, $y(6)$, $y(7)$, $y(10)$, $y(15)$, $y(19)$, $y(21)$, $y(24)$, $y(27)$, …, in Fig. 2 [26].

For convenience, we define the integer sequence $\{t_s : s = 0, 1, 2, \ldots\}$ satisfying

$$0 = t_0 < t_1 < t_2 < t_3 < \cdots < t_{s-1} < t_s < \cdots,$$

with $t_s^* := t_{s+1} - t_s \geq 1$. Thus $y(t)$ is available only when $t = t_s$ ($s = 0, 1, 2, \ldots$), namely, the set $\{y(t_s): s = 0, 1, 2, \ldots\}$ contains all available outputs and the unavailable data $y(t_s+1)$, $y(t_s+2)$, …, $y(t_{s+1}-1)$ are all missing for all $s = 0, 1, 2, \ldots$. For instance, for the scarce measurement pattern in Fig. 2, $y(t_i)$, $i = 0, 1, 2, \ldots$, are available for $t_0 = 0$, $t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 7$, $t_5 = 10$, $t_6 = 15$, $t_7 = 19$, $t_8 = 21$, $t_9 = 24$, $t_{10} = 27$, …. This is a general framework in which we assume the patterns with scarce output availability; of course, it includes all output availability as special cases when $t_s^* = 1$ for all s. If t_s^* is a constant for all s, say, $t_s^* = q$ (a positive integer), we obtain a dual-rate system with input–output sampling ratio equal to q.

For such a scarce output availability pattern in Fig. 2, if the state $\boldsymbol{x}(t)$ in (1) and (2) is measured, then the identification of system parameter matrix/vectors $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ is very easy and they can be estimated from available $u(t)$ and $\boldsymbol{x}(t)$ and scarce $y(t_s)$ using the least squares method like in [22].

This paper considers the case with scarce states $\boldsymbol{x}(t_s)$ being available and the case with all the states being completely unavailable, so the objective is as follows:

1. *The case with scarce states $\boldsymbol{x}(t_s)$ being available*:
   (a) Study the least squares based parameter identification method to estimate the system parameter matrices $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ from the inputs $u(t)$, the available scarce states $\boldsymbol{x}(t_s)$ and the scarce outputs $y(t_s)$.
   (b) Discuss the observer based parameter identification method to estimate the system parameter matrices $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$. The basic idea is using an observer to construct the unavailable states $\boldsymbol{x}(t)$ $(t \neq t_s)$ from the inputs $\{u(t)\}$, the scarce $\{y(t_s)\}$ and $\{\boldsymbol{x}(t_s)\}$.
2. *The case with the states being completely unavailable*: Study the combined parameter and state estimation method for estimating the system parameters and unmeasurable states $\boldsymbol{x}(t)$ based on the canonical state space models only from the available inputs $\{u(t)\}$ and scarce outputs $\{y(t_s)\}$, assuming that the system states $\boldsymbol{x}(t)$ are completely unavailable.

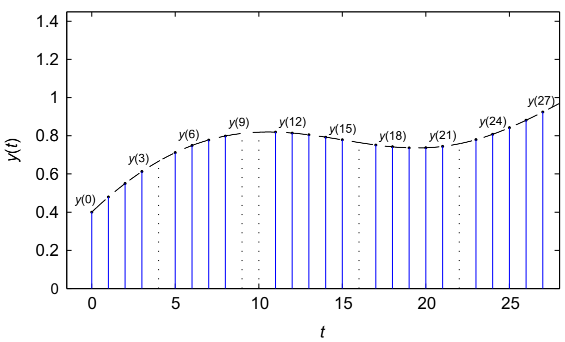

Fig. 1. The missing output data pattern.

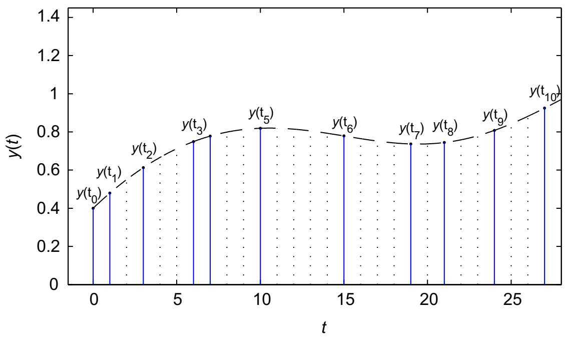

Fig. 2. The scarce measurement pattern.

3. The least squares based parameter identification

Let us introduce some notations first.

Symbols	Meaning
$\mathbf{0}$	the zero matrix of appropriate sizes
$\mathbf{1}_{m \times n}$	an $m \times n$ matrix whose entries are all 1