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Robust time–frequency representation based on the signal normalization and concentration measures

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ABSTRACT

An efficient procedure for obtaining time–frequency representations under high influence of impulsive noise is proposed in this paper. The procedure uses the fast Fourier transform based algorithm instead of sorting procedures common in the case of various robust time–frequency representations proposed recently. Concentration measure is used to select a free parameter of the transform.

1. Introduction

Spectral analysis of nonstationary signals with the high-frequency content corrupted by an impulsive noise has become a very interesting research topic in the past decade. Several filters robust in both time and frequency domains have been proposed [1–6]. Commonly, all these techniques require consuming sorting or iterative procedures for each instant, or for all frequencies, or even for each point in the time–frequency (TF) plane. In this paper, we propose an alternative simple strategy based on the signal normalization. Signal normalization is used as a processing tool for signals corrupted by impulse noise [7–11]. Normalization strategies are already considered in the TF analysis. The fractional lower order technique is introduced in [12]. In addition, a similar technique has been proposed for the TF analysis from the quantum mechanics perspective [13], and used as instantaneous frequency estimator in [14]. Here, we consider an alternative normalization strategy inspired mainly by the problem of inverse filtering in the digital image processing [15]. The proposed normalization strategy has a free parameter. A technique for selecting this parameter, based on the concentration measure for obtaining TF representation (image) of nonstationary signals, is proposed as well.

The paper is organized as follows. Some of the existing techniques for TF imaging of nonstationary signals corrupted by impulsive noise are reviewed in Section 2. The proposed technique is described in Section 3. Numerical examples are given in Section 4, while Section 5 concludes the paper.

2. Spectral analysis of signals corrupted by impulsive noise

Consider a noisy frequency modulated (FM) signal

$$x(t) = A\exp(j\phi(t)) + \nu(t), \tag{1}$$

where $\nu(t)$ is a white noise that can be impulsive and/or heavy tailed. Under these terms it is assumed that the

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noise can take values whose amplitudes are of a significantly higher magnitude than the signal magnitude $|A|$. Common techniques for removing impulsive noise, such as median-, L- or myriad-based filters, are not efficient in FM signal filtering [15,2] since they have low pass characteristics. Their application removes high frequency components from the signal. Therefore, research for alternative techniques becomes a hot issue recently.

Particularly useful techniques are weighted median and myriad filters admitting negative weights [1,2]. They are designed with the same logic as classical linear weighted filters. However, they require a sorting or an iterative procedure to be performed for each instant of the signal. This group of filters is sometimes referred to as the robust filters of FM signals in the time domain.

Another technique is based on the robust DFT evaluation [5,16]. The robust DFT is calculated for each frequency using nonlinear techniques eliminating impulses and providing estimate of the standard DFT of the FM signal. Again these techniques assume sorting or iterative procedures meaning that they are significantly more demanding than the standard (fast) DFT based techniques.

Similar problems arise in the case of the TF representations. There are several alternatives how to evaluate the TF representations of signals corrupted by impulsive noise [3,12,17–19]. One possibility is to calculate the robust short time Fourier transform (STFT) for each point (pixel) in the TF plane based on the robust DFT algorithms and to use it for obtaining other robust TF representations [18]. Namely, the higher-order TF representations can be realized by using the STFT calculated in the initial stage, avoiding undesired effect such as the cross-terms. Also, evaluation of the robust TF representation by filtering auto-correlations of the signal is more sensitive to impulse noise errors than the signal (or modulated signal) itself [18].

In general, a robust STFT can be expressed as

$$STFT_R(t,\omega) = \mathrm{R}\{x(t+n\Delta t)\exp(-j\omega n\Delta t) | n \in [-N/2, N/2)\}, \tag{2}$$

where R{} is the robust operator applied on the modulated signal sequence, Δt is the sampling period, while N is the number of samples in the considered window. In the following we are presenting some of the robust STFT forms.

The robust STFT evaluated using the marginal-median approach is defined as

$$\begin{aligned} STFT_{|e|}(t,\omega) = {} & \mathrm{median}\{\mathrm{Re}\{x(t+n\Delta T) \\ & \times \exp(-j\omega(n\Delta t)) | n \in [-N/2, N/2)\} \\ & + j\,\mathrm{median}\{\mathrm{Im}\{x(t+n\Delta T) \\ & \times \exp(-j\omega(n\Delta t)) | n \in [-N/2, N/2)\}. \end{aligned} \tag{3}$$

To calculate the STFT given by (3), one has to preform the sorting procedure for each (t,ω) pair.

The myriad-based STFT can be evaluated using the following iterative procedure:

$$STFT_\kappa^{(i)}(t,\omega) = \frac{\sum_{n=-N/2}^{N/2-1} \frac{x(t+n\Delta t)\exp(-j\omega n\Delta t)}{[\kappa^2 + |x(t+n\Delta t)\exp(-j\omega n\Delta t) - STFT_\kappa^{(i-1)}(t,\omega)|^2]}}{\sum_{n=-N/2}^{N/2-1} \frac{1}{[\kappa^2 + |x(t+n\Delta t)\exp(-j\omega n\Delta t) - STFT_\kappa^{(i-1)}(t,\omega)|^2]}}. \tag{4}$$

with a properly selected initial iteration.

Various robust TF forms can be obtained from the robust STFT as the initial signal representation. Here, we consider the S-method (SM) [20]

$$SM_R(t,\omega) = |STFT_R(t,\omega)|^2 + 2\mathrm{Re}\left\{\sum_{l=1}^{L} STFT_R(t,\omega+l\Delta\omega) STFT_R^*(t,\omega-l\Delta\omega)\right\},$$

where $2L+1$ is the frequency window length and $\Delta\omega$ is the difference between two consecutive samples on the frequency grid in the TF plane. For $L=0$, the robust SM is equal to the robust spectrogram (square magnitude of STFT), while for $L \to N/2$, the robust Wigner distribution (WD) is obtained. Relatively small $L \in [1,10]$ significantly improves concentration of the spectrogram but without cross-terms that appear in the WD. In the same way the SM can be extended to other higher-order TF representations, since all of them can be realized using the STFT [18]. Evaluation of the robust TF representation requires a computationally efficient form of the STFT robust to the impulsive/heavy tailed noise in the initial step. This implies an alternative to the sorting or iterative procedures in order to obtain both fast and accurate TF representations of signals corrupted by impulsive/heavy tailed noise.

3. Signal normalization

Consider the following normalized signal:

$$y(t) = \frac{x(t)}{|x(t)|}. \tag{5}$$

When $x(t)$ is a monocomponent non-noisy signal, (5) becomes $y(t) = \exp(j\phi(t))$ and has the same instantaneous frequency (IF) $\omega(t) = \phi'(t)$ as the original signal $x(t)$, without a proper information on the signal amplitude. However, usually the IF of a signal is more important feature. Based on the IF we can later estimate the signal amplitude. One particular strange situation is that this normalization for signal with varying amplitude will give signal with unit amplitude. In the case of a signal corrupted by a Gaussian noise, the analysis of resulting noise after normalization (5) is performed in [8,9]. For the signal-to-noise ratio $SNR > 5$ dB we cannot expect big difference in the resulting SNR with respect to the signal $x(t)$. Samples that are not corrupted by impulsive noise are proportional to the original samples. Thus we have a similar situation like in the additive Gaussian noise case. Samples corrupted by impulses have unitary amplitude with random phase (probably $\pm\pi/2$ or 0). Variance of this noise is proportional to the percentage of impulses [21]. This noise has no impulsive characteristics.

However, the primary concern in the application of the signal normalization comes in the case of multicomponent signals. Consider a two component signal

$$x(t) = A_1\exp(j\phi_1(t)) + A_2\exp(j\phi_2(t)). \tag{6}$$

Amplitude of $x(t)$ is equal to

$$|x(t)| = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1(t) - \phi_2(t))}. \tag{7}$$

For example, for $A_1 = A_2$ and $\phi_1(t) - \phi_2(t) = (2k+1)\pi$, where $k \in Z$, the obtained amplitude is equal to zero. The normalization in this case is even not possible or it is giving