Signal Processing 104 (2014) 437–449

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Comparative simulation study of fast heuristics for power control in copper broadband networks

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ARTICLE INFO

Article history:
Received 11 November 2013
Received in revised form
28 April 2014
Accepted 1 May 2014
Available online 10 May 2014

Keywords:
Digital subscriber lines
Power control
Meta-heuristics

ABSTRACT

The data-rate in currently deployed multi-carrier digital subscriber line (DSL) communication systems is limited by the interference among copper lines. This interference can be alleviated by multi-user transmit power allocation. Problem decomposition results in a large number of per-subcarrier problems. Our objective is to solve these nonconvex integer per-subcarrier power control problems at low complexity. For this purpose we develop ten combinatorial heuristics and test them by simulation under a small complexity budget in scenarios with tens of DSL users, where optimal solutions are currently intractable. Simulation results lead us to the conclusion that simple randomized greedy heuristics extended by a specific local search perform well despite the stringent complexity restriction. This has implications on multi-user discrete resource allocation algorithms, as these can be designed to *jointly* optimize transmit power among users even in large-scale scenarios.

1. Introduction

In 2012 over 360 million customers world-wide utilized digital subscriber lines (DSL), making it the most widely deployed fixed broadband access technology [1]. While each user has at least one dedicated copper line, multi-carrier DSL systems still suffer from the electromagnetic coupling (or "interference") among the twisted copper pairs. The effect is noise at the receiver, which limits the achievable data-rate and increases the energy consumption per transmitted data-bit. We study the nonconvex integer multi-user problem of controlling the power levels transmitted by all users on the subcarriers in order to lower interference, increase the number of transmitted data-bits, and reduce the transmit-power consumption (implicitly lowering the system power consumption [2]). The objective is formulated as a weighted sum of users' transmit powers and bits, constrained by technology and regulatory restrictions. This fundamental problem also finds applications in wireless networks [3–6].

The main contribution of this study lies in the comparative simulation experiments including ten combinatorial heuristics for large-scale discrete single-subcarrier power control with a *low complexity budget*. We propose a mixture of deterministic and stochastic heuristics, the latter comprising more direct applications of meta-heuristics as well as greedy schemes modified based on problem insights. These insights are derived from an analysis of suboptimality of these deterministic greedy algorithms, and motivate for example randomization-based modifications. Preliminary simulation results have appeared in [7,8]. However, the present study specifically focusses on the single-subcarrier problem and additionally provides full descriptions of all proposed heuristics, their parameter settings, as well as the detailed simulation

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http://dx.doi.org/10.1016/j.sigpro.2014.05.004
0165-1684/

results on an extensive set of 6 and 30-user DSL scenarios. While previously stochastic search heuristics may have been considered too complex for power control on a large number of subcarriers, our results show that simple randomized greedy heuristics enhanced by first-improving local search perform well even in case of a very stringent complexity budget, and that the heuristics' average suboptimality is implicitly dependent on the targeted data-rate.

We begin in Sections 2, 3.1, and 3.2 by reviewing some relevant literature, stating our notation, and reviewing the common nonconvex integer optimization model for power control in DSL, respectively. The “no free lunch” theorem states that, under specific assumptions on the problem domain and applied algorithms, any two algorithms have the same mean performance when compared over all possible problems [9] – we refer to [9] for a precise statement of this result. Hence, optimization heuristics cannot be regarded as superior in general, but should rather be evaluated for the specific problem domain based on a representative problem set. Generic optimization heuristics evaluated on an insufficient problem set may hence perform very differently in practice than what the simulation of the problem set would have suggested. Therefore problem-specific insights shall be exploited in the design of optimization heuristics, and a representative sampling of problem instances is needed in order to judge the performance of heuristics in a specific problem domain. Consequently, we introduce the three generic search principles applied throughout the paper in Section 4, review three basic search schemes from [10] following these search principles in Section 4.1, and in Section 4.2 analyze their performance on a selection of 84 medium-scale problems where optimal solutions are tractable. A branch-and-bound algorithm and a general-purpose mixed-integer nonconvex problem solver [11,12] are applied to generate the optimal reference solutions. It turns out that a specific problem feature (the well-known “near–far problem”) leads to a high suboptimality of two greedy base-heuristics. The insights gained thereby as well as general meta-heuristics are applied in Section 5 to complement the set of, in total, ten combinatorial search heuristics for power control. For example, in Section 5.1 the greedy decisions of loading bits are randomized, while in Section 5.2.1 the *sequence* of greedy per-user decisions is randomized. The simulation setup and the methodology for the parameterization and performance evaluation of the heuristics in medium-scale and large-scale problems are described in Sections 6.1–6.4, respectively. The discussion of results in Section 6.5 is followed by our conclusions in Section 7.

2. Background on power control in DSL

Multi-carrier power control can be modeled as a multi-dimensional nonlinear Knapsack problem [13] which has motivated the application of Lagrange decomposition [14,15]. Thereby one obtains a large number of integer per-subcarrier subproblems which have to be solved numerous times for updating the Lagrange multipliers, and re-optimized when the DSL network changes. This motivates the tight complexity budget we impose upon their solution, where for reproducibility we opt for the number of function evaluations as our complexity metric, cf. Section 6.1 for its relation to simulation time. We explicitly focus on these subproblems only, where the objective is a weighted sum of data-bits and transmit power, cf. the large applicability of this problem [10,15]. Optimal algorithms are only tractable for medium-scale networks with few-users [11,16,17]. However, the low interference coupling compared to the direct coupling over a subscriber line that is typical for DSL [18] makes us believe that simple stochastic search heuristics may perform near-optimal even when constrained to a low number of function evaluations. Efficient state-of-the-art methods for power control in large-scale DSL networks solve a continuous relaxation only or violate the integer bit-loading restriction by iterating over users [19,20]. The nonlinear Dantzig–Wolfe decomposition approach in [10] allows for suboptimal solutions of the per-subcarrier problems, making the application of combinatorial search schemes as dual heuristics attractive. Furthermore, combinatorial heuristics could be applied on top of any previously proposed (e.g., continuous) power allocation algorithm in order to improve a rounded preliminary solution.

3. System and optimization model

3.1. System model and notation

We consider a communication network with U DSL users, and refer to [18] for a more detailed description of the technical background on DSL. By adequate modulation techniques the frequency bandwidth is subdivided into C regularly spaced and mutually exclusive frequency subcarriers. The index sets of users and subcarriers are $\mathcal{U}=\{1,\ldots,U\}$ and $\mathcal{C}=\{1,\ldots,C\}$, respectively. Our optimization variables are the power levels p_u^c applied by user u and subcarrier c. The achievable number of data-bits per channel-access of user u on subcarrier c is modeled as a, in general, nonconvex function $r_u^c(\mathbf{p}^c)$ [bits] [18] that depends on the transmit powers of all users. The transmit powers and data-bits over all users on subcarrier c are more compactly written as $\mathbf{p}^c \in \mathcal{R}_+^U$ and $\mathbf{r}^c(\mathbf{p}^c) \in \mathcal{R}_+^U$, respectively. The inverse function $\mathbf{p}^c(\mathbf{r}^c)$ for a bit allocation $\mathbf{r}^c$ is uniquely computable [21] by solving a system of linear equations of size $U \times U$. A publicly available tool for DSL performance evaluation can be found in [22]. We refer to Section 6.1 for a description of the network scenarios considered in this work.

Transmit power levels are constrained by a regulatory power mask constraint $p_u^c \le \hat{p}_u^c$, $\forall u \in \mathcal{U}$, $c \in \mathcal{C}$, preventing excessive disturbance among competing DSL operators. Furthermore, the implicit constraint $r_u^c(\mathbf{p}^c) \in \mathcal{B}, \forall u \in \mathcal{U}$, $c \in \mathcal{C}$, is motivated by practical modulation schemes which only support an integer number of data-bits in the set $\mathcal{B}=\{1,\ldots,\hat{B}\}$, with a technology-dependent maximum number of data-bits $\hat{B}$ per subcarrier. The feasible set of transmit powers is summarized as $\mathcal{Q}^c=\{\mathbf{p}^c | r_u^c(\mathbf{p}^c) \in \mathcal{B}, 0 \le p_u^c \le \hat{p}_u^c, \forall u \in \mathcal{U}\}$, $c \in \mathcal{C}$. Our objective is defined as the weighted sum of transmit powers and data-bits