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Complex-valued sparse reconstruction via arctangent regularization ☆

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ABSTRACT

Complex-valued sparse reconstruction is conventionally solved by transforming it into real-valued problems. However, this method might not work efficiently and correctly, especially when the size of the problem is large, or the mutual coherence is high. In this paper, we present a novel algorithm called the arctangent regularization (ATANR), which can handle the complex-valued problems of large size and high mutual coherence directly. The ATANR is implemented with the iterative least squares (IRLS) framework, and accelerated by the dimension reduction and active set selection steps. Further, we summarize and analyze the common properties of a penalty kernel which is suitable for sparse reconstruction. The analyses show that the key difference, between the arctangent kernel and the ℓ_1 norm, is that the first order derivative of ATANR is close to zero for a nonzero variable. This will make ATANR less sensitive to the regularization parameter λ than ℓ_1 regularization methods. Finally, lots of numerical experiments validate that ATANR usually has better performance than the conventional ℓ_1 regularization methods, not only for the random signs ensemble, but also for the sensing matrix with high mutual coherence, such as the resolution enhancement case.

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1. Introduction

Sparse reconstruction has been attracting more and more attention in recent decades, especially after the establishment of compressive sensing (CS) by David L. Donoho et al. during 2004–2006 [1–3]. CS employs the ℓ_0 quasi-norm to depict the sparsity of a signal, and describes the sparse reconstruction problem (SRP) as an ℓ_0 quasi-norm optimization, which is proven to be NP-hard [4]. Encouragingly, Candes et al. propose the famous restricted isometric property (RIP) [5], which describes an equivalent condition between the ℓ_1 regularization and the ℓ_0 quasi-norm optimization. The advantages are inspiring: on the one hand, the ℓ_1 norm optimization is convex, thus its local minima is also its global minima; on the other hand, there are already lots of excellent algorithms solving ℓ_1 regularization problems efficiently, such as the least absolute shrinkage and selection operator (LASSO) [6–8], and the ℓ_1 regularized least squares (ℓ_1LS) [9]. Besides the ℓ_1 regularization, many other penalty methods were proposed, including the MC+ algorithm [10], and the SparseNet [11], etc.

It is worth noting that these methods are most originally developed for the real-valued SRPs, and are almost not for complex-valued ones directly. However, in some applications, we really have to handle the complex-valued SRPs. For example, in radar imaging, the desired scattering coefficients are always considered to be complex numbers. In order to solve the complex-valued problems, it is common to transform them into the real-valued ones [12]. However,

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this way will dramatically increase the computation load as the dimensions of the measurement matrix $\mathbf{A} \in \mathbb{C}^{n \times m}$ grow up, and it might fail when the mutual coherence is high.

Although there are a few methods which could deal with the complex-valued problems directly, such as the orthogonal matching pursuit (OMP) [13], the LASSO [6–8], and the sparsity driven method [14]. However, their disadvantages are obvious: For OMP, it shows bad performance when the mutual coherence of $\mathbf{A}$ is high. For LASSO, it is time consuming for searching a proper regularization parameter λ by the cross validation method [15]. Even for the LARS-LASSO [8] which does not need to assign a proper λ, it might be still very slow when n and m are both large. Because it requires to compute the whole solution paths first, and then select a proper solution by AIC, BIC or C_p-type risk [8,7,15]. Moreover, when $n \ll m$ and the mutual coherence is high, the LARS-LASSO might fail to find a proper positive direction during its iterations. For sparsity driven method, it has good performance on enhancing the features of block targets; however, it has several parameters which should be well designed. Besides, the Hilbert transform based methods are usually used to analyze the nonlinear and non-stationary complex signals, such as [16,17]. However, they do not emphasize the sparsity of a signal, so that they show worse performance on SRPs than the sparse reconstruction methods.

In this paper, we design an algorithm called the arctangent regularization, which could handle the complex-valued sparse reconstruction problems directly and efficiently, and be suitable for the problem of large size. It is based on the penalty method with the arctangent function as its penalty kernel. Compared with the ℓ_1 norm penalty kernel, the penalty kernel of ATANR is closer to the ℓ_0 quasi-norm. With respect to the ℓ_1 norm penalty kernel, the larger magnitude entry will correspond to a larger penalty term. Whereas the arctangent penalty kernel will suppress the influence of the large magnitudes, such that the solution of ATANR seems to be less sparse than that of ℓ_1 regularization. By the dimension reduction and active set selection steps, ATANR is extended to solve the SRPs of large size. Numerical experiments show that ATANR costs much less execution time than LASSO and ℓ_1LS when the size of the problem reaches 2000×4000.

The remaining sections are organized as follows: In Section 2, we briefly introduce the complex-valued sparse reconstruction problem and some existing penalty algorithms. In Section 3, ATANR is proposed. It is implemented by the IRLS framework, and further improved by the dimension reduction and active set selection steps. In Section 4, the common properties of penalty functions, which are suitable for sparse reconstruction, are summarized and analyzed in detail. These properties expose that the key difference, between ATANR and ℓ_1 regularization, is that the first order derivative of ATANR is close to zero when the variable is nonzero. This difference also results in its less sensitivity to the regularization parameter λ. In Section 5, we focus on the performance of ATANR on the random signs ensemble [18]. Numerical experiments show that ATANR has nearly the same performance as OMP, and outperform LASSO and ℓ_1LS. In Section 6, plenty of simulations were performed for the resolution enhancement case, and ATANR exhibited good performance both in the discrete scatters case and in the continuous block case. In Section 7, we summarize the main work of this paper, and list the next work of ATANR in the future.

2. Complex-valued sparse reconstruction and penalty methods

In this section, we introduce the complex-valued sparse reconstruction problem, and briefly review some excellent existing penalty algorithms.

2.1. Complex-valued sparse reconstruction

The complex-valued sparse reconstruction usually solves an underdetermined linear system

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

where $\mathbf{A} \in \mathbb{C}^{n \times m}$ is the measurement matrix with its rank n and $n \ll m$. $\mathbf{x} \in \mathbb{C}^m$, $\mathbf{y} \in \mathbb{C}^n$ and $\mathbf{n} \in \mathbb{C}^n$ denote the input signal, the measurements and the noise vector, respectively. In CS, the sparsity of $\mathbf{x}$ is defined by the ℓ_0 quasi-norm, namely $\|\mathbf{x}\|_0 = |\mathrm{supp}(\mathbf{x})| = |\{i: \mathbf{x}_i \neq 0\}|$, where $\mathbf{x}_i$ denotes the ith entry of $\mathbf{x}$. When $\mathbf{x}$ has $s \in \mathbb{Z}_+$ nonzero entries, we say $\mathbf{x}$ is s-sparse.

Then, CS describes the SRP as an ℓ_0 quasi-norm optimization.

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0, \quad \text{s.t. } \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \le \varepsilon \tag{1}$$

where ε is related to the variance of the noise vector $\mathbf{n}$, and $\|\bullet\|_2$ denotes the ℓ_2 norm.

Usually, in order to solve the complex-valued SRP, it is required to transform it into the real-valued problem, as shown in (2). It implies that the input signal $\mathbf{x}$ should be transformed into a vector composed of real values, then the linear system $\mathbf{y}=\mathbf{A}\mathbf{x}$ can be rewritten as

$$\begin{bmatrix} \mathbf{A}^r & -\mathbf{A}^i \\ \mathbf{A}^i & \mathbf{A}^r \end{bmatrix} \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^i \end{bmatrix} = \begin{bmatrix} \mathbf{y}^r \\ \mathbf{y}^i \end{bmatrix} \tag{2}$$

It is evident that (2) is real-valued, but meanwhile it increases the problem's dimensions. This indicates that the computation complexity also grows up sharply, thus it is not suitable for the measurement matrix $\mathbf{A}$ of large size. Therefore, we should develop the algorithm handling the complex-valued problems straight and being suitable for the large size problems.

2.2. Penalty methods

The complex-valued problem (1) could be usually solved by the penalty methods, such as the ℓ_1 regularization (LASSO and ℓ_1LS) and the MC+. Generally, the penalized problem usually has a form of

$$\hat{\mathbf{x}}(\lambda) = \arg\min_{\mathbf{x}} L(\mathbf{A}\mathbf{x}, \mathbf{y}) + \sum_{i=1}^{m} J(|x_i|, \lambda_i) \tag{3}$$

where $L(\mathbf{A}\mathbf{x}, \mathbf{y})$ is the fidelity constraint. In most applications, it is considered to be

$$L(\mathbf{A}\mathbf{x}, \mathbf{y}) = \tfrac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$$

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