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A modified frequency-domain block LMS algorithm with guaranteed optimal steady-state performance

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The bin-normalized frequency-domain block LMS (FBLMS) algorithm has low computational burden and potential fast convergence; however, it suffers from a biased steady-state solution when the reference signal lags behind the desired signal or the adaptive filter is of insufficient length. This paper proposes a unified framework for the FBLMS algorithm, which can be used to comprehensively analyze its steady-state behavior. Furthermore, a modified FBLMS algorithm with guaranteed optimal steady-state performance is proposed based on the framework. Simulations are carried out to demonstrate the benefit of the proposed algorithm.

1. Introduction

Adaptive filtering has been widely used in many situations such as telecommunication systems, acoustic echo cancellation, active noise control, and array processing, where the least-mean-square (LMS) algorithm is commonly used due to its simplicity and robustness [1,2]. Unfortunately, it suffers from slow convergence for reference signals with large eigenvalue disparity, and moreover, its computational burden is too heavy in many application scenarios because the filter length has to be set very large [1,2].

To overcome the problem of slow convergence, transform-domain LMS (TDLMS) algorithms [1–4] have been suggested, which preprocess the reference signal by using orthogonal transforms such as the discrete Fourier transform (DFT), discrete cosine transform (DCT), discrete sine transform (DST), and discrete Hartely transform (DHT), and then set power-normalized step sizes. The improvement of the convergence rate has been proven by many researchers [3,4]. However, the computational burden of the TDLMS algorithms is substantially heavier than that of the LMS algorithm because the orthogonal transforms are often performed for each new input sample. Although partial updating and sliding transform techniques [5,6] can be used to mitigate the problem, the computational burden is still a challenge for implementation of the TDLMS algorithms in real-time systems.

Apart from the application of the TDLMS algorithm, the DFT in particular can also be used to realize the frequency-domain block least-mean-square (FBLMS) algorithm [7], which is a computational efficient implementation of the block LMS (BLMS) algorithm. The computational burden of the FBLMS algorithm is significantly less than that of the LMS algorithm because the fast Fourier transform (FFT) is used to calculate both the block filtering output and the update terms in the frequency domain. Furthermore, when the step size of the adaptive filter is normalized by the reference signal power in each frequency bin, the convergence speed of the FBLMS algorithm can be significantly increased for reference signals with large power spectral disparity [7,8]. Therefore the bin-normalized FBLMS algorithm is widely used in many applications that require

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large filter length and fast convergence, e.g., acoustic echo cancellation, active noise control, channel estimation, and equalization [9–11]. Nevertheless, it has been pointed out that the bin-normalized FBLMS algorithm suffers from an increase in the steady-state mean-square error in non-causal circumstances [12] or with insufficient filter length [13], which is very common in many applications of adaptive filters. For example, the reference signal can lag behind the desired signal in an adaptive equalizer or in an adaptive feedback active noise control system. On the other hand, for an acoustic echo cancellation systems used in a room with long reverberation time, it is often the case that the adaptive filter is of insufficient length. A frequency-domain Newton's algorithm has been derived in [12] to improve the steady-state behavior in non-causal circumstances. However, it requires a spectral factorization of the estimated power spectral density of the reference signal, which forms an obstacle to its implementation.

In this paper, a unified framework of the FBLMS algorithm without any assumptions on the signal and system model is proposed, which can be used to comprehensively analyze the steady-state behavior of the algorithm. Based on this framework, a modification is proposed on the existing algorithm that guarantees optimal steady-state behavior. Throughout this paper, lowercase letters are used for scalar quantities, bold lowercase for vectors, and bold uppercase for matrices. Subscript f denotes frequency-domain representation of each signal and k is reserved for the block index.

2. Analysis of FBLMS steady-state behavior based on a unified framework

Let $\mathbf{x}(k)=[x(kN-N), x(kN-N+1), \ldots, x(kN+N-1)]^T$ be the reference signal vector, where the superscript T represents the transpose operation, $\mathbf{w}(k)=[w_0(k), w_1(k), \ldots, w_{N-1}(k)]^T$ be the N-tap filter, and $\mathbf{d}(k)=[d(kN-N), d(kN-N+1), \ldots, d(kN+N-1)]^T$ be the desired signal vector. Then the error vector in the frequency domain can be described as

$$\mathbf{e}_f(k)=\mathbf{F}\mathbf{G}_{0,N}\mathbf{F}^{-1}\left[\mathbf{d}_f(k)-\mathbf{X}_f(k)\mathbf{w}_f(k)\right] \tag{1}$$

where $\mathbf{F}$ represents the $2N\times 2N$ discrete Fourier transform (DFT) matrix, $\mathbf{d}_f(k)=\mathbf{F}[\mathbf{0}_{1\times N}, \mathbf{d}^T(k)]^T$, $\mathbf{X}_f(k)=\text{diag}[\mathbf{x}_f(k)]=\text{diag}[\mathbf{F}\mathbf{x}(k)]$, $\mathbf{w}_f(k)=\mathbf{F}[\mathbf{w}^T(k), \mathbf{0}_{1\times N}]^T$, and

$$\mathbf{G}_{0,N}=\begin{bmatrix}\mathbf{0}_{N\times N} & \mathbf{0}_{N\times N}\\ \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N}\end{bmatrix}. \tag{2}$$

There are two kinds of FBLMS algorithms: constrained and unconstrained [7]. The unconstrained FBLMS algorithm is more computationally efficient by removing the constrained operations; however, the aliasing caused by circular convolution leads to poorer convergence behavior [8]. Therefore this paper focuses only on the constrained algorithm.

The constrained filter update equation in the frequency domain is given by [7]

$$\mathbf{w}_f(k+1)=\mathbf{w}_f(k)+\mathbf{F}\mathbf{G}_{N,0}\mathbf{F}^{-1}\mu\mathbf{M}_f\mathbf{X}_f^H(k)\mathbf{e}_f(k) \tag{3}$$

where the superscript H represents the conjugate transpose operation, μ is a constant step size, $\mathbf{M}_f=\text{diag}[\xi]$ is a diagonal matrix with ξ representing a vector containing the normalizing factors for each frequency bin, and

$$\mathbf{G}_{N,0}=\begin{bmatrix}\mathbf{I}_{N\times N} & \mathbf{0}_{N\times N}\\ \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N}\end{bmatrix}. \tag{4}$$

Note that the non-causal part of the filter coefficients is not affected by the updating process, due to the constraint operation in (3). Therefore, multiplying both sides of (3) by $\mathbf{F}^{-1}$ yields

$$\begin{bmatrix}\mathbf{w}(k+1)\\ \mathbf{0}_{N\times 1}\end{bmatrix}=\begin{bmatrix}\mathbf{w}(k)\\ \mathbf{0}_{N\times 1}\end{bmatrix}+\mu\mathbf{G}_{N,0}\mathbf{M}\mathbf{X}(k)\begin{bmatrix}\mathbf{0}_{N\times 1}\\ \mathbf{e}(k)\end{bmatrix}, \tag{5}$$

where $\mathbf{e}(k)=[e(kN), e(kN+1), \ldots, e(kN+N-1)]^T$,

$$\mathbf{X}(k)=\mathbf{F}^{-1}\mathbf{X}_f^H(k)\mathbf{F}=\begin{bmatrix}\mathbf{X}_1 & \mathbf{X}_2\\ \mathbf{X}_2 & \mathbf{X}_1\end{bmatrix} \tag{6}$$

is a circulant matrix whose first row is $\mathbf{x}(k)$, and

$$\mathbf{M}=\mathbf{F}^{-1}\mathbf{M}_f\mathbf{F}=\begin{bmatrix}\mathbf{M}_1 & \mathbf{M}_2\\ \mathbf{M}_2 & \mathbf{M}_1\end{bmatrix} \tag{7}$$

is also a circulant matrix whose first column is $\mathbf{F}^{-1}\xi$ (the inverse Fourier transform of the normalizing vector).

With simple derivation, (5) becomes

$$\mathbf{w}(k+1)=\mathbf{w}(k)+\mu[\mathbf{M}_1\mathbf{X}_2+\mathbf{M}_2\mathbf{X}_1]\mathbf{e}(k) \tag{8}$$

where

$$\mathbf{e}(k)=\mathbf{d}(k)-\mathbf{X}_2^T\mathbf{w}(k). \tag{9}$$

Taking expectation on both sides of (8) and using the independence assumption with respect to the reference signal and filter coefficients [1,2] yields

$$E\left[\mathbf{w}(k+1)\right]=\left[\mathbf{I}_{N\times N}-\mu\mathbf{M}_1\mathbf{R}-\mu\mathbf{M}_2\hat{\mathbf{R}}\right]E\left[\mathbf{w}(k)\right]+\mu\mathbf{M}_1\mathbf{r}+\mu\mathbf{M}_2\hat{\mathbf{r}}, \tag{10}$$

with

$$\begin{aligned}\mathbf{R}&=E\left[\mathbf{X}_2\mathbf{X}_2^T\right]=N\mathbf{R}_x\\ \hat{\mathbf{R}}&=E\left[\mathbf{X}_1\mathbf{X}_2^T\right]\\ \mathbf{r}&=E\left[\mathbf{X}_2\mathbf{d}(k)\right]=N\mathbf{r}_{dx},\\ \hat{\mathbf{r}}&=E\left[\mathbf{X}_1\mathbf{d}(k)\right],\end{aligned} \tag{11}$$

where $\mathbf{R}_x$ represents the autocorrelation matrix of the reference signal and $\mathbf{r}_{dx}$ represents the correlation vector between the reference signal and the desired signal, and both of these are needed for the Wiener solution. The steady-state solution of (10) is

$$E\left[\mathbf{w}_\infty(k)\right]=\left[\mathbf{M}_1\mathbf{R}+\mathbf{M}_2\hat{\mathbf{R}}\right]^{-1}\left[\mathbf{M}_1\mathbf{r}+\mathbf{M}_2\hat{\mathbf{r}}\right]. \tag{12}$$

Eq. (12) is a unified description without any assumption on the signal and system models, based on which, the steady-state behavior of the FBLMS algorithm can be investigated.

If a constant normalizing factor is used in the frequency domain, i.e., $\mathbf{M}_f=\xi\mathbf{I}_{2N\times 2N}$, then $\mathbf{M}_2=\mathbf{0}_{N\times N}$ according to (7), so that

$$E\left[\mathbf{w}_\infty(k)\right]=\mathbf{R}^{-1}\mathbf{r}=\mathbf{R}_x^{-1}\mathbf{r}_{dx}, \tag{13}$$

which is exactly the causal Wiener filter [1,2].