



Fast Communication

A cumulant-based approach for direction finding in the presence of mutual coupling

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ABSTRACT

To estimate the direction-of-arrivals (DOAs) of non-Gaussian signals impinging on a uniform linear array (ULA) with unknown mutual coupling, a fourth-order cumulants (FOC)-based approach is presented. A series of FOC matrices are constructed by taking advantage of the banded symmetric Toeplitz structure of the mutual coupling matrix (MCM). It is shown that the DOAs can be estimated in a closed form based on these matrices. Compared with the conventional FOC-based method which only uses the middle subarray, the proposed approach makes use of the whole array and it is able to achieve better performance. Moreover, in the proposed approach the DOAs are determined by the eigenvalue decomposition instead of the exhaustive grid search and hence, it is computationally efficient. Finally, compared with the middle subarray method, the proposed one is applicable to scenarios with stronger mutual coupling. A number of numerical examples are provided to validate essential performance improvements achieved by means of the proposed approach.

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1. Introduction

The problem of direction finding in the presence of mutual coupling has attracted much attention [1,2]. In [1], a subspace-based approach is developed to iteratively estimate the direction-of-arrivals (DOAs) and unknown mutual coupling coefficients. The other iterative method is also presented in [2]. Generally, these kinds of methods have high computational complexities since a large number of unknown parameters are involved. To overcome this problem, auto-calibration methods with less unknown parameters and lower complexities have been proposed in [3–5]. These methods based on the fact that the effect of mutual coupling between two sensor elements is inversely related to their distance and can be ignored when they are

separated by few wavelengths. For instance, in [3], the middle subarray of the uniform linear array (ULA) is utilized for DOA estimation using the multiple signal classification (MUSIC) algorithm. On the contrary, the whole ULA is employed by a subspace approach in [4,5] to achieve better performance.

It is worth noting that the aforementioned methods are developed based on second-order statistics and cannot perform well in colored Gaussian noise. However, in certain cases such as digital communications, the received signals are non-Gaussian processes, which have valuable statistical information in their higher-order moments (see, for example, [6–8]). Furthermore, higher-order statistics can be used to suppress the effects of colored Gaussian noise [8]. To this end, high-order statistics have been exploited for DOA estimation in the presence of mutual coupling [9,10]. In [9], a FOC-based calibration method is presented for uniform circular array (UCA) with unknown mutual coupling. Then, in [10], the FOC is applied to direction finding in ULAs with unknown mutual coupling. Though promising results as shown in [10] can be obtained, similar to the second-order

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statistics-based method in [3], only the middle subarray is utilized. This implies that the effective array aperture is reduced. The other drawback of this FOC-based method is that it cannot perform satisfactorily or even fail in the case of relatively strong mutual coupling. The main reason is that this method completely relies on the middle subarray, therefore, very few sensors can be utilized if the ULA is highly coupled or the total number of sensors is limited.

Motivated by the shortcomings of the conventional FOC-based method for direction finding in ULAs with unknown mutual coupling, a new FOC-based approach is proposed in this paper. Exploiting the banded symmetric Toeplitz structure of the mutual coupling matrix (MCM), we obtain a series of FOC matrices, which are then employed for DOA estimation. In our proposed method, the DOA estimates can be obtained through eigenvalue decomposition (EVD) without grid search. Thus, compared with the conventional FOC-based method [10] which uses the spectral grid search to determine the DOAs, the proposed method is computationally more efficient. More importantly, in our proposed method, the whole ULA instead of the middle subarray is utilized and hence improved performance can be achieved. Another essential advantage of the proposed method is that it can be applied to ULAs with stronger mutual coupling, to which the conventional FOC-based method may not be applicable.

2. Problem formulation

A ULA with N omnidirectional sensors is considered. In the absence of uncertainties, the steering vector for an angle θ can be expressed as

$$\mathbf{a}(\theta) = [1, \beta(\theta), \dots, \beta(\theta)^{N-1}]^T \quad (1)$$

where $\beta(\theta) = \exp(j2\pi\lambda^{-1}d \sin \theta)$, d is the inter-sensor spacing, λ is the signal carrier wavelength, $j = \sqrt{-1}$, and $(\cdot)^T$ denotes the transpose operation. However, in the presence of mutual coupling, the steering vector should be written as [3–5]

$$\bar{\mathbf{a}}(\theta) = \mathbf{C}\mathbf{a}(\theta) \quad (2)$$

where \mathbf{C} denotes the MCM composed of the mutual coupling coefficients. Because of the symmetric geometry of the ULA, the MCM can be modeled as a symmetric Toeplitz matrix [1–5]. Furthermore, it is known that the effect of mutual coupling between two sensors is inversely related to the distance, therefore, it is negligible when these two sensors are separated by a certain distance. Assume that the mutual coupling is ignorable when the sensors are separated by P or more inter-sensor spacings, then the MCM can be represented by the following banded symmetric Toeplitz matrix [3–5]:

$$\mathbf{C} = \begin{bmatrix} 1 & c_1 & \cdots & c_{P-1} & \cdots & \mathbf{0} \\ c_1 & 1 & c_1 & \cdots & \ddots & \\ \vdots & c_1 & 1 & c_1 & \cdots & c_{P-1} \\ c_{P-1} & \cdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \cdots & c_1 & 1 & c_1 \\ \mathbf{0} & \cdots & c_{P-1} & \cdots & c_1 & 1 \end{bmatrix}. \quad (3)$$

It should be noticed that in certain cases, e.g., the inter-sensor spacing is very small, the effect of mutual coupling among the

sensors will become strong. Therefore, the above simplification has to be made with a larger P . Furthermore, it is worth mentioning that the MCM may not be a banded symmetric Toeplitz matrix in some situations. However, this direction falls out of the scope of this paper and is an interesting topic for future research.

Based on the above model and assume that K independent zero-mean non-Gaussian narrowband signals impinge on the ULA from far-field with unknown directions $\theta_1, \theta_2, \dots, \theta_K$, the observed array output can be written as

$$\mathbf{x}(t) = \sum_{k=1}^K \bar{\mathbf{a}}(\theta_k) s_k(t) + \mathbf{n}(t) = \bar{\mathbf{A}}\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (4)$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)]$ and $\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1), \bar{\mathbf{a}}(\theta_2), \dots, \bar{\mathbf{a}}(\theta_K)]$ are the $N \times K$ nominal and actual steering matrices, respectively. $\mathbf{s}(t)$ is the vector of signal waveforms as $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$, $\mathbf{n}(t)$ represents the zero-mean Gaussian noise vector which may have arbitrary covariance matrix $E[\mathbf{n}(t)\mathbf{n}^H(t)]$. $E[\cdot]$ and $(\cdot)^H$ denote the mathematical expectation and the Hermitian transpose, respectively.

In order to determine the DOAs from the array observations, a FOC-based method has been proposed in [10]. In this method, the $P-1$ sensors on both sides of the ULA are treated as auxiliary elements. Hence, only the output of the rest middle subarray is utilized and the number of sensors in this middle subarray is given by

$$\tilde{N} = N - 2P + 2. \quad (5)$$

It is shown in [10] that an MUSIC-like approach can be applied to find the DOAs based on the FOC matrix of the output of the middle subarray. However, as mentioned earlier, a variety of problems may arise in this method. First, it is computationally expensive because of the grid search. Second, only the middle subarray is utilized and hence the effective aperture is reduced. At last, the number of sensors in the middle subarray is determined by (5), therefore, in the case of strong mutual coupling, i.e., P is large, \tilde{N} will be very small. Similar problem may also happen when N is not sufficiently large. This implies that only very few sensors in the center of the array can be used by this method. For instance, if $P=N/2$ and $K=4$, this middle subarray method is not applicable. This is because in this case one gets $\tilde{N}=2$ and hence there are only three different entries in the virtual steering vector (i.e., $\bar{\mathbf{a}}(\theta) \otimes \bar{\mathbf{a}}^*(\theta)$ in Eq. (16), [10]) and at most two DOAs can be estimated. Motivated by these problems, a new FOC-based approach is presented in the following section.

3. Proposed FOC-based approach

Following our recent work in [5], it is known that the steering vector (2) can be parameterized as follows:

$$\bar{\mathbf{a}}(\theta) = \mathbf{C}\mathbf{a}(\theta) = \tau(\theta)\mathbf{\Gamma}(\theta)\mathbf{a}(\theta) \quad (6)$$

where

$$\tau(\theta) = 1 + \sum_{i=1}^{P-1} c_i(\beta(\theta)^i + \beta(\theta)^{-i}) \quad (7)$$

and it is assumed to be nonzero. $\mathbf{\Gamma}(\theta)$ is an $N \times N$ diagonal matrix as

$$\mathbf{\Gamma}(\theta) = \text{diag}\{\mu_1 \cdots \mu_{P-1} \ 1 \cdots 1 \ \alpha_1 \cdots \alpha_{P-1}\} \quad (8)$$

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