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Variable regularization for normalized subband adaptive filter

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ABSTRACT

To overcome the performance degradation of least mean square (LMS)-type algorithms when input signals are correlated, the normalized subband adaptive filter (NSAF) was developed. In the NSAF, the regularization parameter influences the stability and performance. In addition, there is a trade-off between convergence rate and steady-state mean square deviation (MSD) according to the change of the parameter. Therefore, to achieve both fast convergence rate and low steady-state MSD, the parameter should be varied. In this paper, a variable regularization scheme for the NSAF is derived on the basis of the orthogonality between the weight-error vector and weight vector update, and by using the calculated MSD. The performance of the variable regularization algorithm is evaluated in terms of MSD. Our simulation results exhibit fast convergence and low steady-state MSD when using the proposed algorithm.

1. Introduction

Adaptive filters are widely used in many applications such as system identification, prediction, and equalization [1,2]. The normalized least-mean-square (NLMS) algorithm is the most popular adaptive filter algorithm because it is simple and robust. However, this algorithm has slow convergence for correlated input signals. To overcome this drawback, the normalized subband adaptive filter (NSAF) algorithm was developed in [3]. The NSAF algorithm partitions the full-band input signals into subband input signals, which are close to white signals in each subband. Therefore, the NSAF exhibits fast convergence for correlated input signals. In addition, the computational complexity of the NSAF algorithm is similar to that of the NLMS algorithm [3].

Similar to the NLMS algorithm, the stability and performance of the NSAF algorithm depends on the regularization parameter [1,2]. The regularization parameter, which is a positive constant, is added to the squared Euclidean norm of the input signal to solve numerical problems when the power of the input signal is close to zero [1]. Any fixed regularization parameter reflects a trade-off between convergence rate and steady-state mean-square deviation (MSD) [4–6]. To overcome this problem, several variable regularization methods have been introduced for the NSAF [5,6]. The steady-state MSD of the variable regularization matrix (VRM) for the NSAF algorithm, which is called the NSAF-VRM in [6], is smaller than that in [5].

In this paper, we propose a new variable regularization scheme for the NSAF that employs calculated MSD to determine the variable regularization, which is derived by using the orthogonality between the weight-error vector and weight vector update. The rationale of the proposed algorithm is to decrease the regularization parameter for large MSD to achieve fast convergence, and

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increase the regularization parameter for small MSD to achieve low steady-state MSD. Therefore, the proposed algorithm has both fast convergence and low steady-state MSD. The simulation results illustrate that the steady-state MSD of the proposed algorithm is smaller than the NSAF-VRM algorithm [6].

The remainder of this paper is organized as follows. In Section 2, we briefly introduce the NSAF algorithm and propose a new variable regularization scheme for the NSAF. Simulation results for the proposed algorithm are presented in Section 3. Section 4 presents our conclusion.

2. Proposed algorithm

2.1. Conventional NSAF algorithm

The desired response $d(n)$ is derived from the following model:

$$d(n)=\mathbf{w}_{\text{opt}}^{T}\mathbf{u}(n)+\eta(n), \tag{1}$$

where $\mathbf{w}_{\text{opt}}$ is an unknown $M\times 1$ vector to be estimated, $(\cdot)^T$ denotes vector or matrix transpose, $\mathbf{u}(n)=[u(n),u(n-1),\ldots,u(n-M+1)]^T$ is the input vector, $\eta(n)$ represents the system noise, which is a zero-mean white Gaussian noise and variance σ_η^2, and M is the unknown system length. Fig. 1 shows the configuration of the NSAF. Signals $d(n), u(n)$, and $\eta(n)$ are partitioned into $d_i(n), u_i(n)$, and $\eta_i(n)$ by the analysis filters, $H_i(z), i=0,1,\ldots,N-1$, where N is the number of subbands. By decimating $d_i(k)$ and $y_i(k)$, we obtain $d_{i,\text{D}}(k)$ and $y_{i,\text{D}}(k)$, which is defined as $y_{i,\text{D}}(k)=\hat{\mathbf{w}}(k)^T\mathbf{u}_i(k)$, where $\mathbf{u}_i(k)=[u_i(k),u_i(k-1),\ldots,u_i(k-M+1)]^T$ and $(\cdot)_\text{D}$ denotes a decimation, respectively.

The update equation of the NSAF algorithm is [3]

$$\hat{\mathbf{w}}(k+1)=\hat{\mathbf{w}}(k)+\mu\sum_{i=0}^{N-1}\frac{\mathbf{u}_i(k)e_{i,\text{D}}(k)}{\|\mathbf{u}_i(k)\|^2}, \tag{2}$$

where μ is the step size parameter and $\|\cdot\|$ represents the Euclidean norm.

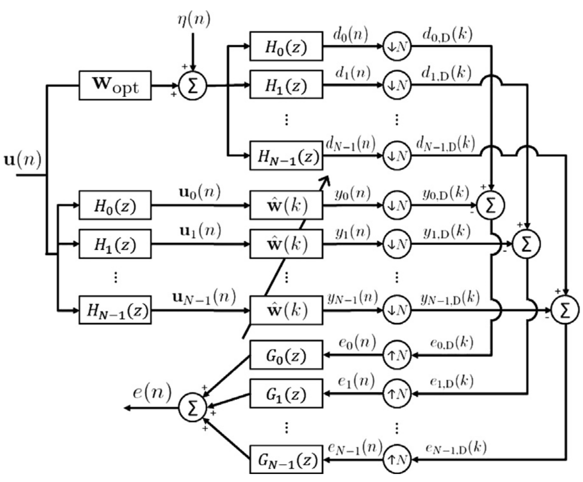

Fig. 1. Configuration of the NSAF.

2.2. Proposed algorithm

The matrix version of the update equation for the NSAF is [5,6]:

$$\hat{\mathbf{w}}(k+1)=\hat{\mathbf{w}}(k)+\mu\mathbf{U}(k)\mathbf{\Lambda}^{-1}(k)\mathbf{e}_\text{D}(k), \tag{3}$$

where

$$\mathbf{U}(k)=[\mathbf{u}_0(k),\mathbf{u}_1(k),\ldots,\mathbf{u}_{N-1}(k)], \tag{4}$$

$$\mathbf{\Lambda}(k)=\mathbf{\Phi}(k)+\mathbf{\Delta}(k), \tag{5}$$

$$\mathbf{\Phi}(k)=\text{diag}[\|\mathbf{u}_0(k)\|^2,\|\mathbf{u}_1(k)\|^2,\ldots,\|\mathbf{u}_{N-1}(k)\|^2], \tag{6}$$

$$\mathbf{\Delta}(k)=\text{diag}[\delta_0(k),\delta_1(k),\ldots,\delta_{N-1}(k)], \tag{7}$$

$$\mathbf{e}_\text{D}(k)=[e_{0,\text{D}}(k),e_{1,\text{D}}(k),\ldots,e_{N-1,\text{D}}(k)]^T, \tag{8}$$

and $\delta_i(k)$ is the variable regularization parameter for the ith subband. The variable regularization parameter for the NSAF algorithm is derived by using MSD, which is defined as $\text{MSD}(k)\triangleq E\{\tilde{\mathbf{w}}^T(k)\tilde{\mathbf{w}}(k)\}$, where the weight-error vector is defined as $\tilde{\mathbf{w}}(k)=\mathbf{w}_{\text{opt}}-\hat{\mathbf{w}}(k)$. By subtracting (3) from $\mathbf{w}_{\text{opt}}$, we obtain

$$\tilde{\mathbf{w}}(k+1)=\tilde{\mathbf{w}}(k)-\mu\mathbf{U}(k)\mathbf{\Lambda}^{-1}(k)\mathbf{e}_\text{D}(k). \tag{9}$$

The weight vector update is defined as

$$\begin{aligned}\delta\hat{\mathbf{w}}(k)&\triangleq\frac{1}{\mu}[\hat{\mathbf{w}}(k+1)-\hat{\mathbf{w}}(k)],\\&=\mathbf{U}(k)\mathbf{\Lambda}^{-1}(k)\mathbf{e}_\text{D}(k).\end{aligned} \tag{10}$$

We rearrange (9) in terms of $\delta\hat{\mathbf{w}}(k)$:

$$\tilde{\mathbf{w}}(k+1)=\tilde{\mathbf{w}}(k)-\mu\delta\hat{\mathbf{w}}(k). \tag{11}$$

By using square of Euclidean norm and the expectation of (11), we get

$$\text{MSD}(k+1)=\text{MSD}(k)-2\mu E\{\tilde{\mathbf{w}}^T(k)\delta\hat{\mathbf{w}}(k)\}+\mu^2E\{\|\delta\hat{\mathbf{w}}(k)\|^2\}. \tag{12}$$

From (12), the convergence condition, $\text{MSD}(k+1)-\text{MSD}(k)\le 0$, is guaranteed if and only if $-2\mu E\{\tilde{\mathbf{w}}^T(k)\ \delta\hat{\mathbf{w}}(k)\}+\mu^2E\{\|\delta\hat{\mathbf{w}}(k)\|^2\}\le 0$. By using the orthogonality between $\tilde{\mathbf{w}}(k+1)$ and $\delta\hat{\mathbf{w}}(k)$, this condition is always satisfied and $-2\mu E\{\tilde{\mathbf{w}}^T(k)\delta\hat{\mathbf{w}}(k)\}+\mu^2E\{\|\delta\hat{\mathbf{w}}(k)\|^2\}$ has minimum value. Therefore, we obtain

$$E\{\tilde{\mathbf{w}}^T(k)\delta\hat{\mathbf{w}}(k)\}=\mu E\{\|\delta\hat{\mathbf{w}}(k)\|^2\}. \tag{13}$$

We make the following assumptions: $\eta_{i,\text{D}}(k), \mathbf{u}_i(k)$, and $\tilde{\mathbf{w}}(k)$ are mutually independent [1]; the ith subband input signal is similar to a white signal [2,8], i.e., $E\{\mathbf{u}_i^T(k)\mathbf{u}_i(k)\}\approx M\sigma_{u_i}^2(k)$ and $E\{\mathbf{u}_i(k)\mathbf{u}_i^T(k)\}\approx\sigma_{u_i}^2(k)\mathbf{I}_M$, where $\sigma_{u_i}^2(k)\triangleq E\{\|u_i(kN)\|^2\}$, and $\mathbf{I}_M$ is the $M\times M$ identity matrix; the ith subband input energy $\|\mathbf{u}_i(k)\|^2$ has the small fluctuation from one to the next [1]; and the subband signals are uncorrelated, such that the ith subband signals are not dependent on signals outside the ith subband [2]. By using the uncorrelated assumption, (13) is rewritten as

$$E\left\{\frac{\tilde{\mathbf{w}}^T(k)\mathbf{u}_i(k)e_{i,\text{D}}(k)}{\|\mathbf{u}_i(k)\|^2+\delta_i(k)}\right\}=\mu E\left\{\frac{\|\mathbf{u}_i(k)e_{i,\text{D}}(k)\|^2}{[\|\mathbf{u}_i(k)\|^2+\delta_i(k)]^2}\right\},\quad i=0,1,\ldots,N-1. \tag{14}$$

By using the above assumption and the approximation that $\delta_i(k)$ is a deterministic value by nature [6], both sides