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Review

Recursive hidden input estimation in nonlinear dynamic systems with varying amounts of *a priori* knowledge

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ABSTRACT

Estimation of additive driving-forces (e.g., hidden inputs) in nonlinear dynamic systems is addressed with varying amounts of *a priori* knowledge on system models exemplified by three typical scenarios: (1) there is no sufficient prior knowledge to build a mathematical model of the underlying system; (2) the system is partially described by an analytic model; (3) a complete and accurate model of the underlying system is available. Three algorithms are proposed for each scenario and analyzed comprehensively. The adaptive driving-force estimator (ADFE) [1,2] is used for the retrieval of driving-forces using only the system outputs for the first scenario. A variational Bayesian and a Bayesian algorithm are established for the second and the third scenarios, respectively. All three algorithms are studied in depth on a nonlinear dynamic system with equivalent computational resources, and the Posterior Cramer–Rao Lower Bounds (PCRLB) are specified as performance metrics for each case. The results lead to a thorough understanding of the capabilities and limitations of the ADFE, which manifests itself as an effective technique for the estimation of rapidly varying hidden inputs *unless* a complete and accurate model is available. Moreover, the methods developed in this paper facilitate a suitable framework for the construction of new and efficient tools for various input estimation problems. In particular, the proposed algorithms constitute a readily available basis for the design of novel input residual estimators to approach the Fault Diagnosis and Isolation (FDI) problem from a new and different perspective.

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1. Introduction

Estimation of hidden states, retrieval of driving-forces (e.g., hidden inputs and external perturbations), or detection of hidden parameters of a dynamic system is typically an ill-posed inverse problem. The availability of sufficient prior knowledge on the underlying phenomenon can greatly facilitate solving such inverse problems. This enables one to invoke Bayesian inference to find regularized solutions to inverse problems by virtue of incorporating all prior knowledge on the system in a probabilistically optimum fashion. However, in many other cases, (e.g., voice production, brain activity, sea dynamics, a wireless multipath link, and evolution of seismic waves) there may be too little or no prior knowledge and evidence to build a specific and accurate model of the underlying system for use of the Bayesian methods. The requirement in such cases is to reconstruct the quantities of interest using only a finite set of time series data without the availability of an analytic model. As a consequence, finding regular solutions to such inverse problems is a common need for many applications in physical, engineering, and biomedical systems.

Our interest is in the estimation of the driving-forces that perturb nonlinear dynamic systems. Those perturbation signals can be viewed as unknown inputs to the unperturbed dynamics, which are hidden from the observer most of the time if not always. A few examples of how the driving-forces in many physical phenomena affect the system dynamics are the mobility of transmit and/or receive terminals in a wireless channel, the effect of wind or small random elements on the sea surface for the evolution of the sea clutter, blinking of eye during the measurement of brain activity, and the development of abrupt or incipient faults in a nonlinear control system. The evolution of brain activity and the development of faults in nonlinear dynamic systems are some good examples of how the internal complexity of some dynamic systems could make it virtually impossible to devise an accurate model of the system.

1.1. Literature review

Estimation of hidden inputs has received considerable attention in both engineering [3–26] and physics [27–35] disciplines. In what follows, we present a brief summary of the vast amount of literature in three categories with respect to the assumed availability of *a priori* knowledge on the system.

1.1.1. Input estimation in fully identified systems

Most of the engineering literature for the input estimation in linear systems has focused on the fault diagnosis and isolation, where abrupt or incipient faults are treated as hidden inputs and inferred via parity check methods [3–5], or through the generalizations of the linear optimum Gaussian filtering (i.e., Kalman filter) [6–8].

For the estimation of hidden inputs in fully identified nonlinear deterministic systems, a commonly employed approach is to transform the nonlinear dynamics to a new set of coordinates where linear methods can be applied [9–14]. The literature on the input estimation in nonlinear stochastic systems is typically concerned with the estimation of static or slowly changing variables by augmenting the state model for incorporation of hidden parameters. This allows the use of sequential Markov Chain Monte Carlo methods (MCMC) (or particle filters) that operate on an augmented system model [15–22]. In a recent work, authors addressed the maximum *a posteriori* probability (MAP) estimates of state and hidden input jointly using Gauss–Newton method [23].

1.1.2. Input estimaton in partially known systems

All the methods described in [3–23] assume that a complete and accurate analytic system model is available. However, in some cases, the unknown system can be specified only partially, where the missing parts of the model need to be inferred from data. A typical approach for the estimation of missing model parts is embodied by parameterizing the missing part, and employing the missing data-maximum likelihood (ML) methods. To this end, the Expectation-Maximization (EM) method is proven to be an effective method [36,24] for parameter estimation in partially known systems. In [25,26], authors introduce a mixture of Gaussian (moG) model for the unknown measurement equation in a nonlinear dynamic system, and estimate the parameters of the moG model so as to accomplish the desired state and input estimation.

1.1.3. Input estimation in unknown systems

Estimation of hidden inputs in nonlinear systems with unknown dynamics has received attention mostly from