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Robust weighted fusion Kalman filters for multisensor time-varying systems with uncertain noise variances

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ABSTRACT

This paper addresses the design of robust weighted fusion Kalman filters for multisensor time-varying systems with uncertainties of noise variances. Using the minimax robust estimation principle and the unbiased linear minimum variance (ULMV) optimal estimation rule, the five robust weighted fusion time-varying Kalman filters are presented based on the worst-case conservative systems with the conservative upper bounds of noise variances. The actual filtering error variances or their traces of each fuser are guaranteed to have a minimal upper bound for all the admissible uncertainties of noise variances. A Lyapunov equation approach is presented to prove the robustness of the robust Kalman filters. The concept of robust accuracy is presented and the robust accuracy relations among the local and fused robust Kalman filters are proved. Specially, the corresponding steady-state robust local and fused Kalman filters are also presented for multisensor time-invariant systems, and the convergence in a realization of the local and fused time-varying and steady-state Kalman filters is proved by the dynamic error system analysis (DESA) method and dynamic variance error system analysis (DVESA) method. A simulation example is given to verify the robustness and robust accuracy relations.

1. Introduction

Multisensor data fusion has been received great attention in recent years and widely applied in many fields including, tracking, communications, signal processing and GPS positioning. The main aim of data fusion is how to combine the local measurement data or local state estimators of multisensor systems to obtain a fused state estimator, whose accuracy is higher than that of each local state estimator [1]. There exist two kinds of fusion methods. The first one is the centralized fusion method where all measured sensor data are communicated to the fusion center, and the obtained state estimator is globally optimal in the sense of unbiased linear minimum variance (ULMV) [2]. Its disadvantage is that the computation and communication burden is larger. The second method is the distributed fusion, which can give the globally optimal [3–5] or suboptimal state estimator [6–8]. Under the ULMV rule, there are three weighted distributed fusion algorithms weighted by matrices, diagonal matrices and scalars respectively [6–8]. Based on the weighted least squares method, two weighted measurement fusion algorithms are also presented [9–13], which have the globally optimality [13]. The distributed fusion method can reduce the computation and communication burden and can facilitate fault detection and isolation more conveniently.

The basic tool of multisensor data fusion is the Kalman filtering method. The Kalman filter is designed based on the assumptions that the system model and noise variances are exactly known. When the model parameters and/or noise variances exist uncertainties, the performance of the Kalman filter may be degraded or an inexact model may cause the filter divergence [14]. This has motivated many studies of designing robust Kalman filters.

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A class of robust Kalman filtering problems is to design a Kalman filter such that its actual filtering error variances or their traces are guaranteed to have a minimal upper bound for all admissible uncertainties [15].

In order to design the robust Kalman filters for the systems with the model parameters uncertainties, two important approaches are the Riccati equation approach [15–18] and the linear matrix inequality (LMI) approach [15,19,20]. The asymptotic properties of the time-varying (finite-horizon) robust Kalman filters and the steady-state (infinite-horizon) robust Kalman filters were investigated in [17,21,22], where the rigorously convergence analysis between the steady-state robust Kalman filter and the time-varying robust Kalman filter was not solved. These designs of robust Kalman filters have the limitation that only the uncertainties of model parameters are considered, while the noise variances are assumed to be exactly known.

So far, the robust Kalman filter for systems with uncertain noise variances are seldom considered [23], and the multisensor information fusion robust Kalman filtering is seldom concerned [24–26], and the robustness of the fusers is not rigorously proved.

In this paper, we consider the problem of the robust Kalman filtering for multisensor time-varying systems with uncertain noise variances. Using the minimax robust estimation principle [23,27,28] and ULMV optimal estimation rule, the local robust time-varying Kalman filters and three fused robust time-varying Kalman filters weighted by matrices, diagonal matrices and scalars are respectively presented based on the worst-case conservative systems with the conservative upper bounds of noise variances. In order to compute the weights, the cross-covariances among the local filtering errors are required. However, in many theoretical and practical applications, the computation of the cross-covariances is very difficult and complicated [29,30], or the cross-covariances are unknown [31–33].

In order to overcome this limitation, based on the covariance intersection (CI) fusion method [31–33], the CI fusion robust time-varying Kalman filter is presented in this paper, it can be obtained by convex combination of the local robust Kalman filters and is converted into an optimization problem of a nonlinear function with constraints. Compared with the above three weighted fusion robust time-varying Kalman filters, it avoids the computation of cross-covariances, and suitable for multisensor systems with unknown variances and cross-covariances of the local filters.

Based on the weighted least squares [34], the robust weighted measurement fusion time-varying Kalman filter is also presented, which is equivalent to the centralized robust Kalman fuser [13].

Furthermore, a Lyapunov equation approach is presented to prove the robustness of the proposed robust Kalman filters, which is completely different from the Riccati equation approach and the LMI approach. The concept of robust accuracy is presented for uncertain systems. The robust accuracy relations of the local and weighted fusion robust Kalman filters are proved. Specially, the corresponding robust local and fused time-varying and steady-state Kalman filters are presented for multisensor time-invariant systems with uncertain noise variances, and their convergence in a realization is rigorously proved by the dynamic error system analysis (DESA) method and the dynamic variance error system analysis (DVESA) method [35,36].

The remainder of this paper is organized as follows: Section 2 gives the problem formulation. The local robust time-varying Kalman filter and the proof of the robustness are presented in Section 3. The five weighted fusion robust time-varying Kalman filters together with their proof of the robustness are given in Section 4. The robust accuracy analysis of the local and weighted fusion robust Kalman filters is presented in Section 5. The robust local and fused steady-state Kalman filters and the convergence analysis are presented in Section 6. Section 7 gives a simulation example. The conclusions are presented in Section 8.

2. Problem formulation

Consider the following multisensor linear discrete time-varying systems with uncertain noise variances:

$$x(t+1)=\Phi(t)x(t)+\Gamma(t)w(t) \tag{1}$$

$$y_i(t)=H_i(t)x(t)+v_i(t), \quad i=1,\cdots,L \tag{2}$$

where t represents the discrete time, $x(t)\in R^n$ is the state, $y_i(t)\in R^{m_i}$ is the measurement of the ith subsystem, $w(t)\in R^r$ is the input noise, $v_i(t)\in R^{m_i}$ is the measurement noise of the ith subsystem, $\Phi(t)$, $\Gamma(t)$ and $H_i(t)$ are known time-varying matrices with appropriate dimensions. L is the number of sensors.

Assumption 1. $w(t)$ and $v_i(t)$ are uncorrelated white noises with zero means and unknown uncertain actual variances $\overline{Q}(t)$ and $\overline{R}_i(t)$ at time t, respectively, satisfying

$$\overline{Q}(t)\le Q(t), \overline{R}_i(t)\le R_i(t), \quad i=1,\cdots,L, \quad \forall t \tag{3}$$

where $Q(t)$ and $R_i(t)$ are known conservative upper bounds of $\overline{Q}(t)$ and $\overline{R}_i(t)$, respectively, and

$$E\left[\begin{pmatrix} w(t) \\ v_i(t)\end{pmatrix}\begin{pmatrix} w(k) & v_j(k)\end{pmatrix}^T\right]=\begin{bmatrix}\overline{Q}(t) & 0 \\ 0 & \overline{R}_i(t)\delta_{ij}\end{bmatrix}\delta_{tk} \tag{4}$$

where E is the mathematical expectation operator, the superscript T is the transpose. δ_{ij} is the Kronecker δ function, $\delta_{ii}=1, \delta_{ij}=0 (i\ne j)$.

Assumption 2. The initial state $x(0)$ is independent of $w(t)$ and $v_i(t)$ and has mean value μ and unknown uncertain actual variance $\overline{P}(0|0)$ which satisfies

$$\overline{P}(0|0)\le P(0|0) \tag{5}$$

where $P(0|0)$ is a known conservative upper bound of $\overline{P}(0|0)$.

The so-called robust Kalman filtering problem is to design a Kalman filter $\hat{x}(t|t)$ for uncertain system (1) and (2), such that its actual filtering error variances $\overline{P}(t|t)$ or their traces $tr\overline{P}(t|t)$ have a minimal upper bound $P(t|t)$ or $trP(t|t)$ for all admissible uncertainties satisfying (3) and (5) [15], i.e.,

$$\overline{P}(t/t)\le P(t/t) \quad \text{or} \quad tr\overline{P}(t/t)\le trP(t/t) \tag{6}$$