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Fast communication

Design of frequency-invariant robust beam patterns by the oversteering of end-fire arrays

F. Traverso [a], M. Crocco [b], A. Trucco [a,b,*]

[a] Department of Electrical, Electronic, Telecommunications Engineering and Naval Architecture (DITEN), University of Genoa, via Opera Pia 11, I-16145 Genova, Italy
[b] Pattern Analysis & Computer Vision, Istituto Italiano di Tecnologia (IIT), I-16163 Genova, Italy

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ABSTRACT

The end-fire steering of a data-independent beamformer is well suited to achieving superdirective performance by a linear array whose aperture is shorter than the wavelength. Here, we focus on frequency-invariant beam patterns obtained by filter-and-sum beamformers that are robust against errors and fluctuations. We demonstrate that the oversteering technique applied to a weakly directive beam pattern can considerably increase the directivity, providing a frequency invariance that is better than those of traditional methods. The performance is evaluated with respect to the maximum constrained directivity that a given array can provide at the lower bound of the frequency band.

1. Introduction

Array systems aimed at spatially processing broadband signals frequently require that the amplitude of their response at any direction of arrival (DOA) be adequately constant over a wide frequency band. This requirement is quite common in audio signal processing by microphone arrays, where the impinging signals should be attenuated depending on their DOA while avoiding distortion of their spectra. The achievement of a frequency-invariant beam pattern (FIBP) through a filter-and-sum data-independent beamformer [1,2] is more difficult when the spatial aperture is shorter than the involved wavelengths. This situation typically occurs in microphone arrays for hearing aids and mobile sound capture. To attain a directivity of some interest, the design of a superdirective beam pattern (BP) becomes essential [3,4], raising the problem of performance sensitivity to array imperfections. Over the past few decades, several methods have been proposed to design broadband arrays [3,5–7] whose performance is made both superdirective and robust. The basic way to achieve this result is to introduce a constraint on the sensitivity factor [3], i.e., the inverse of the white noise gain (WNG) of the beamformer, and to maximize the constrained directivity. However, although this technique provides optimum design at a given frequency, it is not tailored for the synthesis of robust FIBP.

Broadside steering and end-fire steering represent two classical options for processing far-field waves by means of a linear array with a fixed looking direction. However, end-fire steering has an important advantage: when the inter-element distance is smaller than a half-wavelength, the maximum constrained directivity of a broadside array does not exceed N (N being the number of elements) [8], whereas the constrained directivity of an end-fire array can approach N^2 [9]; when the entire array aperture is smaller than the wavelength, the constrained directivity

* Corresponding author at: DITEN, University of Genoa, via Opera Pia 11, I-16145 Genova, Italy. Tel.: +39 010 353 2253.
E-mail addresses: trucco@ieee.org, trucco@dibe.unige.it (A. Trucco).

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obtainable by an end-fire array may be smaller than N but is significantly higher than that obtainable by a broadside array.

For these reasons, we focus on end-fire arrays with a robust, superdirective FIBP. The common design strategy [5–7,10,11] is based on the minimization of a cost function expressing the distance between the obtained BP and the desired FIBP. Such a desired beam pattern (DBP) has to be set arbitrarily by the user [5–7,11] through a time-consuming, trial-and-error process that does not assure the reaching of the optimal choice. To the best of the authors' knowledge, [10] presents the only method that enables a maximum-directivity FIBP design without the need for setting a priori a DBP. For a given statistic of the array imperfections, the method in [10] allows for tuning the tradeoff between directivity and frequency invariance (FI), with higher directivities yielding poorer FIs. Although it was demonstrated for broadside arrays, such a method can be applied to the synthesis of end-fire arrays as well. We verified that the method in [10] is effective in designing solutions that provide, at the lower bound of the frequency band, the optimum end-fire directivity for a given value of the sensitivity factor. However, for end-fire steering, this method revealed some imperfections in FI that were not encountered for broadside steering.

Here, we propose an alternative method that is specifically tailored for the design of broadband end-fire arrays; the proposed method performs similarly to the method in [10] in terms of directivity and sensitivity factor but has better FI. The method we are proposing employs the concept of oversteering [3,12,13]: the directivity of an end-fire array is increased by introducing additional delays that push the main-lobe peak past end-fire, outside the visible region. In this manner, the main-lobe width is reduced and the directivity is increased, but unfortunately, the sensitivity factor is also increased. Fig. 1 shows an example of the oversteering applied to an end-fire BP, at a given frequency. In the proposed method, a slightly modified version of [10] is used to compute an end-fire solution in which the FI is maximally stressed. Such a solution (referred to as starting solution) is used as a support for the oversteering operation: by varying the oversteering amount, different solutions that hold an optimum balance between directivity and sensitivity factor are obtained. It is possible to verify that such solutions perform well in terms of FI.

This paper is organized as follows. Section 2 summarizes the method proposed in [10] for the generation of FIBPs. Section 3 presents the oversteering of a broadband array and a metric to evaluate the FI. Finally, Section 4 presents some results and Section 5 provides our conclusions.

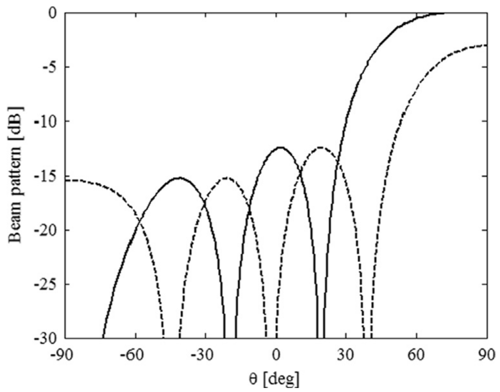

Fig. 1. End-fire BPs for an array of six transducers spaced at a distance $d=\lambda/4$ (where λ is the wavelength) without oversteering (solid line) and with oversteering (dashed line). A uniform weighting window was employed.

2. Filter-and-sum beamforming optimization

Let us consider a linear array composed of N omnidirectional, point-like transducers centered at the coordinate origin and placed on the x axis. When broadband processing is implemented through a filter-and-sum beamformer [1,10] whose FIR filters are composed of L taps each, the ideal BP can be written as follows:

$$B_i(u,f,\mathbf{r}) = \sum_{n=0}^{N-1}\sum_{l=0}^{L-1} r_{n,l} \exp\left[j2\pi f\left(\frac{x_n u}{c} - lT\right)\right] \quad (1)$$

where f is the frequency, x_n is the position of the nth transducer, $u=\sin\theta$ (θ is the angle indicating the DOA, measured with respect to the y-axis), c is the speed of acoustic waves in the medium, T is the sampling interval of the FIR filters, $r_{n,l}$ is a real value representing the lth tap coefficient of the nth filter and the vector $\mathbf{r}$ contains all of the taps $r_{n,l}$. The directivity $D(f)$ of a linear array steered in the direction θ_0 is defined as follows:

$$D(f) = \frac{|B_i(u_0,f,\mathbf{r})|^2}{\left(\frac{1}{2}\right)\int_{-1}^{1}|B_i(u,f,\mathbf{r})|^2\,du} \quad (2)$$

For the same array, the WNG $G(f)$ is defined as follows:

$$G(f) = \frac{|B_i(u_0,f,\mathbf{r})|^2}{\sum_{n=0}^{N-1}|H_n(f)|^2} \quad (3)$$

where $H_n(f)$ is the frequency response of the nth filter.

Because the transducers are not perfectly matched to each other, to compute the actual BP of the array, a complex random variable $A_n = a_n \exp(j\gamma_n)$ can be introduced to model the gain a_n and the phase γ_n of the nth transducer's response

$$B_a(u,f,\mathbf{r}) = \sum_{n=0}^{N-1}\sum_{l=0}^{L-1} r_{n,l}A_n \exp\left[j2\pi f\left(\frac{x_n u}{c} - lT\right)\right] \quad (4)$$

To attain a FIBP, a method was proposed in [10] that allows for the design of a robust broadband beamformer with a tunable tradeoff between the FI and directivity, without the need to impose *a priori* a DBP. The key idea was to perform a global optimization, simultaneously synthesizing the FIR filter-taps producing the optimized FIBP and the values of the DBP. The optimization of the DBP values consists of the maximization of the DBP directivity, whereas the optimization of the filter-taps consists of maximizing the adherence between the obtained BP and the DBP. Therefore, a cost function $J(\mathbf{r}, \mathbf{d})$, representing the weighted sum of the DBP energy[1] and the least-square distance between the obtained BP and DBP, was defined as

[1] The minimization of DBP energy corresponds approximately to the maximization of the DBP directivity [10].