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Optimal design of Hermitian transform and vectors of both mask and window coefficients for denoising applications with both unknown noise characteristics and distortions

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ABSTRACT

This paper proposes an optimal design of a Hermitian transform and vectors of both mask and window coefficients for denoising signals with both unknown noise characteristics and distortions. The signals are represented in the vector form. Then, they are transformed to a new domain via multiplying these vectors to a Hermitian matrix. A vector of mask coefficients is point by point multiplied to the transformed vectors. The processed vectors are transformed back to the time domain. A vector of window coefficients is point by point multiplied to the processed vectors. An optimal design of the Hermitian matrix and the vectors of both mask and window coefficients is formulated as a quadratically constrained programming problem subject to a Hermitian constraint. By initializing the window coefficients, the Hermitian matrix and the vector of mask coefficients are derived via an orthogonal Procrustes approach. Based on the obtained Hermitian matrix and the vector of mask coefficients, the vector of window coefficients is derived. By iterating these two procedures, the final Hermitian matrix and the vectors of both mask and window coefficients are obtained. The convergence of the algorithm is guaranteed. The proposed method is applied to denoise both clinical electrocardiograms and electromyograms as well as speech signals with both unknown noise characteristics and distortions. Experimental results show that the proposed method outperforms existing denoising methods.

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1. Introduction

Noises are usually corrupted to signals. For biomedical applications [16,17], these noises will affect the diagnosis. For audio-applications, these noises will affect the qualities of speeches and cause misunderstanding among people. Hence, denoising plays a very important role in our daily life.

When the characteristics of the noises are known, Wiener filtering [15] can be employed to suppress the noises. However, this approach is not applied if the noises characteristics are unknown. When signals are localized in certain regions of uniformly rectangular time frequency grids, filtering in the frequency domain and windowing in the time domain via short time Fourier transform approaches can significantly suppress the noises outside the signal regions [1–3]. For certain types of signals such as chirp signals, signals are localized in certain regions of rotated time frequency domains [4–6]. For this case, better denoising performances can be achieved if the mask operations are performed in discrete fractional Fourier transformed domains and windowing is applied in the time domain [4–6]. When signals are localized in certain regions of dyadically rectangular time frequency grids, performing either the hard thresholding or the soft thresholding in orthonormal wavelet domains can significantly improve the signal-to-noise ratios [7–9]. In fact, the short time Fourier transform based denoising approaches [1–3], the discrete fractional Fourier transform based denoising approaches [4–6] and the orthonormal wavelet based denoising approaches [7–9] are particular cases of performing mask operations in Hermitian transformed domains and windowing in the time domain [10–12]. Hence, better performances are expected if the optimal mask operations are performed in the optimal Hermitian transformed domains. Nevertheless, there is no result for finding the optimal Hermitian transform and the corresponding optimal vectors of both mask and window coefficients. This motivates us to address this problem.

To perform the optimal mask operation in the optimal Hermitian transformed domain, it is required to design both the optimal Hermitian matrix and the corresponding optimal mask coefficients. This design problem is different from conventional mask coefficient design problems. For the conventional filter design problems, the transform matrix (the discrete Fourier transform matrix) is known because filtering is defined in the frequency domain. In this case, the transform matrix is not required to be designed. This is similar to the case of designing the optimal mask coefficients in a discrete fractional Fourier transformed domain when the rotational angle of the discrete fractional Fourier transform is known. Also, this is similar to the case of designing either the hard thresholding value or the soft thresholding value in an orthonormal wavelet domain when the orthonormal wavelet kernel is predefined. On the other hand, this problem is different from conventional principal component analysis kernel design problems. For the conventional principal component analysis kernel design problems, only the unitary matrix is required to be designed. If the transformed coefficients are kept, the corresponding mask coefficients are one. If the transformed coefficients are discarded, the corresponding mask coefficients are zero. Hence, it is not required to design the mask coefficients. Moreover, the principal component analysis kernel is real valued, while the transformed matrix designed in this paper is complex valued. From these viewpoints, we can see that the problem studied in this paper is the generalization of existing fixed kernel mask coefficient design problems and existing principal component analysis kernel design problems.

Since the Hermitian condition is characterized by a quadratic complex valued matrix equality [10–12], the design problem studied in this paper is actually an optimization problem subject to a quadratic complex valued matrix equality constraint. In general, it is very difficult to find the solutions of such kind of constrained optimization problems. To address this difficulty, an orthogonal Procrustes approach is employed for solving the problem. It is worth noting that both the mask operation in a Hermitian transformed domain and the windowing operation in the time domain are linear. Hence, the overall operation is linear. As a result, instead of optimizing the Hermitian matrix and the vectors of both mask and window coefficients, the overall linear operator is optimized. Once the overall linear operator is optimized, the optimal Hermitian matrix and the corresponding optimal vectors of both mask and window coefficients are determined accordingly.

The outline of this paper is as follows. In Section 2, a signal model and the problem formulation are presented. In particular, the design of the Hermitian matrix and the vectors of both mask and window coefficients is formulated as an optimization problem subject to a quadratic complex valued matrix equality constraint. In Section 3, a method for solving such kind of optimization problems is presented. In Section 4, the proposed method is applied to denoise both clinical electrocardiograms and electromyograms as well as speech signals with both unknown noise characteristics and distortions. Experimental results are presented. Finally, a conclusion is drawn in Section 5.

2. Signal model and problem formulation

Suppose that signals can be represented as N dimensional vectors. Assume that there are M training pairs of original and measured signals. Denote the original signals as $\mathbf{y}_i \in \Re^{N\times 1}$ for $i=0,\ldots,M-1$. Here, $\Re^{a\times b}$ denotes the space of $a\times b$ real valued matrices. Moreover, assume that there is an unknown nonlinear distortion applied to $\mathbf{y}_i \in \Re^{N\times 1}$ for $i=0,\ldots,M-1$. Let the operator characterizing the nonlinear distortion be $\aleph : \Re^{N\times 1} \to \Re^{N\times 1}$ and the nonlinearly distorted signals be $\mathbf{x}_i \in \Re^{N\times 1}$ for $i=0,\ldots,M-1$. That is, $\mathbf{x}_i = \aleph(\mathbf{y}_i)$ for $i=0,\ldots,M-1$. Furthermore, suppose that there is an unknown linear distortion applied to $\mathbf{x}_i \in \Re^{N\times 1}$ for $i=0,\ldots,M-1$. Let the operator characterizing the unknown linear distortion be $\mathbf{H} \in \Re^{N\times N}$. In addition, assume that the linearly distorted signals are further corrupted by additive noises. Let the corrupted noises be $\mathbf{n}_i \in \Re^{N\times 1}$ for $i=0,\ldots,M-1$. Here, it is neither assumed that the noises are uncorrelated to the signals nor the characteristics