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Image reconstruction from a complete set of geometric and complex moments

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A B S T R A C T

An image can be reconstructed from the finite set of its orthogonal moments. Since geometric and complex moment kernels do not satisfy orthogonality criterion, direct image reconstruction using them is deemed to be difficult. In this paper, we propose a technique to reconstruct an image from either geometric moments (GMs) or complex moments (CMs). We utilize a relationship between GMs and Stirling numbers of the second kind. Then, by using the invertibility property of the Stirling transform, the original image can be reconstructed from its complete set of either geometric or complex moments. Further, based on previous works on blur effects on a moment domain and using the proposed reconstruction methods, a formulation is shown to obtain an estimated original image from the degraded image moments and the blur parameter. The reconstruction performance of the proposed methods on blur images is presented to validate the theoretical framework.

1. Introduction

Moment functions computed from an image have been widely used as a basic feature descriptors in image analysis, pattern and object recognition, image classification, degraded signals classification, image watermarking and template matching [1–6]. Two-dimensional geometric moments (GMs), sometimes also called usual moments, were introduced by Hu [7]. He used the theory of algebraic invariants to derive a set of GM functions that are invariant with respect to translation, scaling as well as rotation. These types of moments not only provide measures of the shapes, such as volumes and surface areas, but also allow for the encoding of a shape with descriptors that are amenable to fast analysis such as database screening or pairwise shape comparison.

Complex moments (CMs) were introduced by Abu-Mostafa and Psaltis [8] as a simple and direct method to perform image normalization. However, kernel functions of GMs and CMs are not orthogonal. This makes the reconstruction of an image from its GMs or CMs quite difficult.

Teague [9] has proposed three different approaches to solve the inverse problem of non-orthogonal moments. The first approach was the usage of characteristic function. He showed a relationship between the 2D Fourier transform of an image and its geometric moments by using the expansion of Taylor series. Then from the 2D Fourier transform, the original image can be obtained by the inversion formula. However, the length of order to be used for an accurate reconstruction may exceed the size of the image.

Using the Teagues' first approach, Ghorbel et al. [10] applied the discrete Fourier transform (DFT) as a characteristic function to reconstruct the original image using the approximated expansion of exponential function kernel up to a limited order of GMs. However, by taking the same number of GMs order as size of the image, this approximation causes error [11]. Ghorbel et al. [12] also used the

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DFT approximation method for image reconstruction from a complete set of CMs. They proposed a systematic method to reconstruct an image from a finite set of its moment invariants by exploiting the link between the DFT of an image and its CMs.

Teague's second method for determining an inverse GM transform is based on moment matching. Teague applied a continuous function as a polynomial to reconstruct the original image. This approach seems to be impractical as it requires the solution to an increasing number of coupled equations as higher order moments are considered.

The third approach to overcome the image reconstruction problem from a set of non-orthogonal moments is using orthogonal moments. Teague [9] showed that by replacing the monomial kernel of GMs with Legendre and Zernike continuous orthogonal polynomials, an image can be reconstructed from their orthogonal moments. Recently, a set of another continuous orthogonal moments is introduced such as Gaussian–Hermite moments [13], Bessel–Fourier moments [14] and Gegenbauer moments [15].

Mukundan et al. [16] and Yap et al. [17] introduced a set of discrete orthogonal moments based on the discrete Tchebichef polynomials and Krawtchouk polynomials, respectively. The development of these discrete orthogonal moments spurs the growth of other discrete orthogonal moments such as Hahn, dual-Hahn, Meixner, Charlier and Racah moments [18–21]. If we use the full set of Tchebichef or Krawtchouk moments, the original image can be recovered accurately.

Flusser et al. [11] propose a direct method to reconstruct an image using its computed geometric moments. As reported in [11], the algorithm produces exact reconstruction for images up to 11×11. For larger images, higher order moments as well as the kernel functions lose their precision and make it difficult for the image to be reconstructed exactly.

Though the geometric and complex moments are used as object descriptors, there has been substantial work done to link geometric and complex moments to point spread functions. In this regard, Flusser et al. [11] derived a general expression for degraded image moments in terms of the original image moments and the point-spread function (PSF). Additionally, they also showed a relationship that can be formulated between CMs of the degraded image caused by the Gaussian blur and the original image. In the case of GMs of the degraded image caused by the Gaussian blur, Liu and Zhang [22] derived an explicit formula for the GMs of the PSF.

In this paper, based on their contributions which lack the reconstruction of the image from GMs or CMs, we were motivated to solve the reconstruction directly from GMs and CMs. By using the relationship established in [11,22], we can formulate an inverse relationship between the original image and the blurred image. Firstly, we used the relationship of Stirling numbers [23] of the second kind and the kernels of GMs to generate the GMs. This paves the way to use the invertibility property of the Stirling transform to reconstruct an image from a complete set of GMs. Since CMs can be expressed in terms of GMs, we can also implement the same approach to reconstruct the original image from its CMs. It is worth noting, for an accurate reconstruction, the moments order has to be of the same size as the original image. We then show an application to restore original image from the blurred image using the derived reconstruction capabilities of either GMs or CMs and blur parameter.

This paper is organized as follows. In Section 2, a brief review of the related research and the results obtained are discussed. Section 3 gives the proposed methodology of GMs and CMs reconstruction. In this section, by using the Stirling transform, we show how we derive a new formulation for image reconstruction based on its GMs and CMs. The representation of the blurred image reconstruction using the proposed method is given in Section 4. In Section 5, the computational aspects of the proposed algorithm are outlined through experimental results which show the desirable features of the new method. Finally, Section 6 concludes the paper.

2. A brief review of the related works

Using GM as a basic tool for the purpose of image processing has several advantages over other orthogonal moments. The focus of most of the works has been on the orthogonal moments, but a lot of them have tried to describe the orthogonal moments in terms of GMs as a linear combination. In this section, some of the basic concepts are reviewed, and complementary ones are proposed.

An image is a real discrete 2D function with size $N \times M$. The GM of order (p, q) of an image, $f(x,y)$ in the spatial domain is defined by

$$m_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{M} x^p y^q f(x,y). \tag{1}$$

The CMs of order (p, q) of the same image in the spatial domain are defined by [8]

$$C_{pq} = \sum_{x=1}^{N} \sum_{y=1}^{M} (x+iy)^p (x-iy)^q f(x,y) \tag{2}$$

where $i = \sqrt{-1}$. The relationship between CMs and GMs can be obtained as follows [11]:

$$C_{pq} = \sum_{k=0}^{p} \sum_{l=0}^{q} \binom{p}{k} \binom{q}{l} (-1)^{q-l} i^{p+q-k-l} m_{k+l,p+q-k-l}. \tag{3}$$

The inverse relationship between GMs and CMs can be obtained as follows [11]:

$$m_{pq} = \sum_{k=0}^{p} \sum_{l=0}^{q} \binom{p}{k} \binom{q}{l} \frac{(-1)^{q-l}}{2^{p+q} i^q} C_{k+l,p+q-k-l}. \tag{4}$$

Eqs. (3) and (4) can be achieved from binomial expansion of the complex kernels in (2).

The discrete Fourier transform (DFT) of an image is defined by the following:

$$F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) e^{-i2\pi((ux/N)+(vy/M))}. \tag{5}$$