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A method for initializing free parameters in lattice structure of linear phase perfect reconstruction filter bank ☆

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ABSTRACT

Since the optimization of the free parameters in the lattice structure of linear phase perfect reconstruction filter bank (LPPRFB) is commonly highly nonlinear, the initialization of the free parameters is important before starting the optimization. We systematically study the initialization of the free parameters, following the way that initializes the free parameters in a higher-order LPPRFB by those in a lower-order one. The proposed method is carried out in two steps: formulating the relation between the higher-order and the lower-order LPPRFBs in terms of polyphase matrices, and then implementing the initialization based on the relation. Design examples show that, compared with published LPPRFBs, the filter banks yielded by our method show comparable or even better performance.

1. Introduction

Filter bank has been extensively used in signal processing [1–4], especially linear phase perfect reconstruction filter bank (LPPRFB). The design of LPPRFB can be efficiently carried out via lattice structure [5–20]. Such obtained filter banks include free parameters, which can be optimized to produce better filter banks. Unfortunately, the optimization is usually highly nonlinear and very sensitive to the initial values of the free parameters, and an improper initialization will yield only suboptimal or even impractical filter banks[1]. Therefore, the initialization of the free parameters is important before starting the optimization.

Such issue has been mentioned in the literature but without systematical studies. In the design examples of [21–23], the free parameters are initialized to be constants or initialized randomly. The initial filter bank (i.e. the filter bank corresponding to the initial values of the free parameters) is generally impractical one (rather than practical one), and it is relatively difficult for the subsequent optimization to produce optimized filter bank with optimal performance.

If the free parameters can be initialized such that the initial values correspond to a practical filter bank (e.g. DCT filter bank), it is more possible for the subsequent optimization to approach optimal optimized filter bank. In the design example of [10], Gan et al. initialized the free parameters in an order-four filter bank by those in a Walsh–Hadamard Transform (WHT) filter bank (an order-zero practical filter bank). In [14], Liang et al. initialized the free parameters in a higher-order filter bank by those in a DCT filter bank (also an order-zero practical filter bank). Such action in [10,14] has reflected the way that initializes the free parameters in a higher-order

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[1] A filter bank is classified to be practical if its coding gain, stopband attenuation or other measure is above a threshold t. Notice that the measure mentioned here has been normalized such that greater value corresponds to better performance.

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filter bank by those in a lower-order one. Nevertheless, the initializations in [10,14] were restricted to be used for the case with lower order of zero, and the lower-order filter bank was limited to be WHT or DCT filer bank. Besides, the initialization in [10] was used for LPPRFB limited with pairwise mirror image frequency response, and the increasing order[2] was four (can be generalized to other even numbers but not including odd numbers).

This paper will give a systematical study of the initialization of the free parameters, following the way that initializes the free parameters in a higher-order LPPRFB by those in a lower-order one. We first formulate the relation between the higher-order and the lower-order LPPRFBs in terms of polyphase matrices, and then implement the initialization according to the relation. The lower order in the proposed method is not limited to be zero, the lower-order filter bank is not restricted to be WHT or DCT filter bank, the increasing order can be any integer allowed by an LPPRFB system, and the method can be used for the LPPRFB without the limitation of pairwise mirror image frequency response. As illustrated in the design examples, in contrast to published LPPRFBs, the filter banks yielded by the proposed method show comparable or even better performance. Some notations in this paper are described as follows. Identity matrix, exchange matrix, and null matrix are denoted by $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{0}$, respectively; and the subscripts will be given if their sizes are not clear from the context.

2. Preliminaries

For an M-channel filter bank with each filter of length MK, the analysis filters can be denoted by $H_i(z)=\sum_{n=0}^{KM-1} h_i(n)z^{-n}$, $i=0,\ldots,M-1$, and the analysis polyphase matrix is defined as $\mathbf{E}(z)=(E_{i,l}(z); i,l=0,\ldots,M-1)$ with $E_{i,l}(z)=\sum_{k=0}^{K-1} h_i(Mk+l)z^{-k}$; the synthesis filters can be represented as $F_i(z)=\sum_{n=-KM+1}^{0} f_i(n)z^{-n}$, $i=0,\ldots,M-1$, and the synthesis polyphase matrix is defined as $\mathbf{R}(z)=(R_{i,l}(z); i,l=0,\ldots,M-1)$ with $R_{i,l}(z)=\sum_{k=0}^{K-1} f_l(Mk+i)z^{k}$. Here, $h_i(n)$ and $f_i(n)$ are filter coefficients, and $K-1$ is the order of the system. Such a system is an LPPRFB if and only if the associated polyphase matrices satisfy linear phase and perfect reconstruction properties.

As shown in [24], an LPPRFB only exists in two cases: (1) M is even and K is a positive integer; (2) M is odd and K is an odd positive integer. Based upon [11,25,26], using lattice structure, one can design an LPPRFB by factorizing its polyphase matrix $\mathbf{E}(z)$ as

$$\mathbf{E}(z)=\begin{cases}\mathbf{G}_{K-1}(z)\cdots\mathbf{G}_2(z)\mathbf{G}_1(z)\mathbf{E}_0, & M\in\text{even} \quad (1\text{a})\\ \mathbf{G}_{\frac{K-1}{2}}(z)\cdots\mathbf{G}_2(z)\mathbf{G}_1(z)\mathbf{E}_0, & M\in\text{odd} \quad (1\text{b})\end{cases} \tag{1}$$

where

$$\mathbf{G}_k(z)=\begin{cases}\dfrac{1}{2}\,\mathrm{diag}(\mathbf{I}_m,\mathbf{V}_k)\mathbf{T}_M(z), & M\in\text{even} \quad (2\text{a})\\ \mathrm{diag}(\mathbf{U}_k,\mathbf{I}_m)\mathrm{diag}\big(z^{-1},\tfrac{1}{4}\mathbf{T}_{2m}(z)\\ \quad\cdot\mathrm{diag}(\mathbf{V}_k,\mathbf{I}_m)\mathbf{T}_{2m}(z)\big), & M\in\text{odd} \quad (2\text{b})\end{cases} \tag{2}$$

[2] Increasing order denotes the difference between the higher order and the lower order.

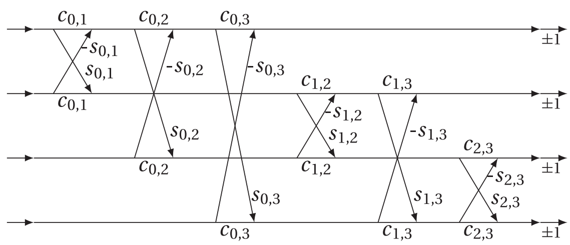

Fig. 1. Factorization of orthogonal matrix.

$$\mathbf{E}_0=\frac{\sqrt{2}}{2}\,\mathrm{diag}(\mathbf{U}_0,\mathbf{V}_0)\mathbf{W}_M\,\mathrm{diag}\big(\mathbf{I}_{M-m},\mathbf{J}_m\big). \tag{3}$$

Here, $m=\lfloor M/2\rfloor$, where $\lfloor x\rfloor$ denotes the floor of the real number x; the free invertible matrices $\mathbf{U}_k$ and $\mathbf{V}_k$ are of size $m+1$ and m, respectively. Besides,

$$\mathbf{T}_{2m}(z)=\mathbf{W}_{2m}\,\mathrm{diag}(\mathbf{I}_m,z^{-1}\mathbf{I}_m)\mathbf{W}_{2m}. \tag{4}$$

The symbol $\mathbf{W}_{2m}$ here as well as $\mathbf{W}_{2m+1}$ denote special matrices as follows:

$$\mathbf{W}_{2m}=\begin{bmatrix}\mathbf{I}_m & \mathbf{I}_m\\ \mathbf{I}_m & -\mathbf{I}_m\end{bmatrix} \tag{5}$$

$$\mathbf{W}_{2m+1}=\begin{bmatrix}\mathbf{I}_m & & \mathbf{I}_m\\ & \sqrt{2} & \\ \mathbf{I}_m & & -\mathbf{I}_m\end{bmatrix}. \tag{6}$$

Based on the perfect reconstruction property (i.e. $\mathbf{R}(z)\mathbf{E}(z)=z^{-n_0}\mathbf{I}$, wherein n_0 is an integer), the factorization of $\mathbf{R}(z)$ can be easily obtained. Hence, we only focus on the analysis polyphase matrix $\mathbf{E}(z)$ in this paper.

The free parameters in the lattice structure of LPPRFB are all included in the free invertible matrices, i.e. $\mathbf{U}_k$ and $\mathbf{V}_k$. Such free parameters contain Givens rotation angles, sign parameters and positive diagonal entries, as described below. Each free invertible matrix of size m, according to singular value decomposition, can be factorized as the product $\mathbf{ASB}$, where $\mathbf{A}$ and $\mathbf{B}$ are free orthogonal matrices, and $\mathbf{S}$ is a positive diagonal matrix. Each free orthogonal matrix of size m (like $\mathbf{A}$ and $\mathbf{B}$), based on Givens rotation decomposition [27, pp. 747–751], can be represented by $m(m-1)/2$ Givens rotation angles and m sign parameters; for example, such a matrix of size four can be expressed as in Fig. 1, where $c_{i,j}=\cos\,\alpha_{i,j}, s_{i,j}=\sin\,\alpha_{i,j}$, $\alpha_{i,j}$ is the Givens rotation angle, and ± 1 represents the sign parameter. Each positive diagonal matrix of size m (like $\mathbf{S}$) can be represented by m positive real numbers.

3. Main result

In the following discussion, the symbols and their version marked with the superscript "′" correspond to an order-$(K-1)$ LPPRFB and an order-$(K'-1)$ one, respectively. We try to initialize the free parameters in the order-$(K-1)$ LPPRFB such that its filter coefficients satisfy

$$\begin{aligned}&(h_i(0),\ldots,h_i(KM-1))\\ &\quad=(\mathbf{0}_{1\times a_0},h_i'(0),\ldots,h_i'(K'M-1),\mathbf{0}_{1\times a_0})\end{aligned} \tag{7}$$

where $a_0=M(K-K')/2$, and $h_i'(n)$ is the filter coefficient of a practical LPPRFB with order $K'-1$, $K'<K$. Here, a_0 is