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A novel covariation based noncircular sources direction finding method under impulsive noise environments

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ABSTRACT

We extend the bearing estimation method for noncircular signals to the impulsive noise scenario which can be modeled as a complex symmetric alpha-stable ($S\alpha S$) process. We define the extended covariation based matrix of the sensor outputs and show that it can be applied with subspace techniques to extract the bearing information from noncircular sources. Comprehensive simulations demonstrate that the proposed direction finding algorithm outperforms the traditional NC-MUSIC algorithm in the presence of a wide range of impulsive noise environments.

1. Introduction

Subspace-based methods have been the dominant techniques for estimating the directions of arrival (DOA) of narrowband sources which impinge on an array of sensors [1–3], i.e., the well known eigendecomposition based MUSIC algorithm is to obtain the high-resolution direction estimates by exploiting the orthogonality between a sample subspace and a DOA parameter-dependent subspace [2]. However, there are two distinct disadvantages in the traditional MUSIC algorithm. Firstly, it is not capable of processing more than $M-1$ noncoherent incoming signals from an array of M sensors. Secondly, it is designed to be limited to extracting the bearing information from complex circular signals; that is, for noncircular signals, it suffers from model mismatch and degrades dramatically in its performance.

In recent years, developing direction finding methods for noncircular signals, e.g., binary phased-shift keying (BPSK), offset-quadrature-phase-shift keying (OQPSK) modulated signals, has aroused plenty of interest through exploiting the inherent potentials of noncircular signals. By conducting eigendecomposition on the extended covariance matrix, in which the conventional covariance function is employed in company with the unconjugated spatial covariance function, for the sensor outputs, Gounon proposed the NC-MUSIC algorithm [4]. In Ref. [5], Chargé improved the computation procedure of NC-MUSIC by using a polynomial rooting technique, and proposed the so-called NC-root-MUSIC algorithm. Haardt [6] and Zoubir [7] extended the classic ESPRIT method and proposed the ESPRIT-like algorithms for noncircular signals. The asymptotic performance of the MUSIC-like algorithms has been investigated in Ref. [8]. In comparison with the traditional MUSIC and ESPRIT methods, the maximum number of sources that can be processed has been increased in all the above algorithms and the estimation accuracy has been improved significantly as well. Moreover, high-order cumulants have been explored and the so-called extended $2q$-MUSIC algorithm has been proposed in Ref. [9] for noncircular sources localization, by which the array aperture has been extended further [9].

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Despite the advantages of these algorithms, they also have limitations. These algorithms have been studied under the assumption of Gaussian or second-order model. The utilization of second order or higher order statistics, e.g., covariance, unconjugated covariance, and higher order cumulants, has been the major methodology of these algorithms by assuming that the additive noise is Gaussian distributed. However, for a fairly common case, it is inappropriate to model the noise as Gaussian. Recent studies show that in some scenarios, sudden bursts or sharp spikes are exhibited at the array outputs which can be characterized as impulsive due to clutter sources such as mountains, forests and sea waves [10–12]. This impulsive component of noise has been found to be significant in many problems, including atmospheric noise and underwater problems such as sonar and submarine communication. Taking these scenarios into account, we are interested in developing direction finding methods for noncircular signals suitable for not only Gaussian but also impulsive noise environments.

Researchers have studied this impulsive nature and shown that the symmetric alpha stable ($S\alpha S$) processes are better models for impulsive noise than the Gaussian processes [13–15]. The real $S\alpha S$ process can be defined by its characteristic function $\varphi(\omega)=\exp(ja\omega-\gamma|\omega|^{\alpha})$. Where α is the characteristic exponent taking values $0<\alpha\le 2$, and γ is the dispersion which is similar to the variance of the Gaussian distribution. Specially, Gaussian processes are stable processes with $\alpha=2$. For these distributions with $\alpha<2$, they present heavier tails than those of Gaussian distribution and possess finite pth-order moments only for $p<\alpha<2$. These processes satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. They arise as limiting processes of sums of i.i.d random variables via the generalized central limit theorem. For details about $S\alpha S$ processes, see Ref. [15], and the references therein.

Due to the lack of finite variance, covariances do not exist on the space of $S\alpha S$ random variables. Instead, the quantity covariation plays a role for $S\alpha S$ random variables analogous to the one played by covariance for Gaussian random variables [16]. In this paper, we address the direction finding problem in the presence of impulsive noise by utilizing the extended covariation based matrix of the sensor outputs for noncircular sources. This paper is organized as follows: In Section 2, we present some necessary information on covariation. In Section 3, we define the problem of interest. In Section 4, we propose a new subspace algorithm based on covariation for noncircular sources. Finally, simulation experiments are presented in Section 5, and conclusions are drawn in Section 6.

The following notations are utilized throughout this paper. The superscripts "T", "H" and "*" denote the transpose, the conjugate transpose, and the conjugate, respectively. $E(\cdot)$, $\|\cdot\|_{\text{Fro}}$, $\Re(\cdot)$ and $\Im(\cdot)$ stand for the expectation, Frobenius norm, and real and imaginary part operators, respectively. $\mathbf{I}_M$ is the $M\times M$ identity matrix.

2. Covariation and its properties

A complex random variable (RV) $X=X_1+jX_2$ is $S\alpha S$ if X_1 and X_2 are jointly $S\alpha S$. Several complex RV's are jointly $S\alpha S$ if their real and imaginary parts are jointly $S\alpha S$ [15]. When $X=X_1+jX_2$ and $Y=Y_1+jY_2$ are jointly $S\alpha S$ with $1<\alpha\le 2$, the covariation of X and Y is defined by

$$[X,Y]_\alpha=\frac{E\{XY^{\langle p-1\rangle}\}}{E\{|Y|^p\}}\gamma_Y \quad 1\le p<\alpha \tag{1}$$

where $Y^{\langle b\rangle}=|Y|^{b-1}Y^*$ and γ_Y is the dispersion of RV, Y given by

$$\gamma_Y^{p/\alpha}=\frac{E\{|Y|^p\}}{C(p,\alpha)} \quad \text{for } 0<p<\alpha \tag{2}$$

with

$$C(p,\alpha)=\frac{2^{p+1}\Gamma(p+2/2)\Gamma(-p/\alpha)}{\alpha\Gamma(1/2)\Gamma(-p/2)}, \tag{3}$$

in which $\Gamma(\cdot)$ is the gamma function defined by

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}\,dt \tag{4}$$

Also, the covariation coefficient of X and Y is defined by

$$\lambda_{X,Y}=\frac{[X,Y]_\alpha}{[Y,Y]_\alpha} \tag{5}$$

and by using (1), it can be expressed as

$$\lambda_{X,Y}=\frac{E\{XY^{\langle p-1\rangle}\}}{E\{|Y|^p\}} \quad \text{for } 1\le p<\alpha \tag{6}$$

Next, we list three main properties of covarition of complex jointly $S\alpha S$ RV's as follows:

P1. The covariation $[X,Y]_\alpha$ is linear in X, that is, if X_1, X_2, Y are jointly $S\alpha S$ then

$$[\beta_1X_1+\beta_2X_2,Y]_\alpha=\beta_1[X_1,Y]_\alpha+\beta_2[X_2,Y]_\alpha \tag{7}$$

for any complex constants β_1 and β_2.

P2. $[X,Y]_\alpha$ possesses the following pseudo-linearity property with respect to Y, that is, if Y_1, Y_2 are independent and X, Y_1, Y_2 are jointly $S\alpha S$, then

$$[X,\xi_1Y_1+\xi_2Y_2]_\alpha=\xi_1^{\langle\alpha-1\rangle}[X,Y_1]_\alpha+\xi_2^{\langle\alpha-1\rangle}[X,Y_2]_\alpha \tag{8}$$

for any complex constants ξ_1 and ξ_2.

P3. If X and Y are independent $S\alpha S$, then $[X,Y]_\alpha=0$.

3. Problem formulation

3.1. Array model

Assume P narrow-band independent, complex isotropic $S\alpha S$ $(1<\alpha\le 2)$ signals with locations $\{\theta_1,\theta_2,\ldots,\theta_P\}$ are impinging on a uniformly linear array of M sensors. The underlying noises are i.i.d. isotropic complex $S\alpha S$ random processes with the same characteristic exponent α as the signals. In addition, we also assume that the real parts and the imaginary parts of the complex $S\alpha S$ noises are independent with each other [4–7]. Using the complex envelope representation, the array output can be expressed as

$$x_m(t)=\sum_{k=1}^{P}a(\theta_k)s_k(t)+n_m(t), \quad m=1,2,\ldots,M \tag{9}$$

where $a(\theta_k)=e^{-j(2\pi/\lambda)(m-1)d\,\sin(\theta_k)}$ is the steering coefficient of the mth sensor toward direction θ_k, λ denotes the