Signal Processing 98 (2014) 96–101

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Successive frequency domain minimization for time delay estimation

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ARTICLE INFO

Article history:
Received 25 April 2013
Received in revised form
18 October 2013
Accepted 23 October 2013
Available online 12 November 2013

Keywords:
Time delays
Delay systems
Discrete Fourier Transform
Frequency domain methods
Successive minimization

ABSTRACT

Estimating time delays for signal alignment is important for many applications. This paper extends a successful frequency domain cost function minimization algorithm capable of estimating time delays to within a fraction of sampling periods. Since the function has a narrow basin of attraction around the global minimum, this method often diverges when initial time delay estimates are not sufficiently close to the desired optimal time delays. We propose a second order successive minimization method with reduced sensitivity to initial guesses. Both an analytic expression for the cost function Hessian matrix and a condition guaranteeing positive-definiteness are presented. This condition facilitates the construction of sequentially modified cost functions whose nested minimization increases the basin of attraction around the global minimum. This successive minimization technique is more robust and yields higher accuracy when compared to the original method and the well-known method of Cross Correlator (CC).

1. Introduction

Time-delay estimation is crucial in many applications such as seismology, radar, sonar and communication systems, and biomedical sciences [1–14]. Most of these methods yield integer multiple estimates of the sampling period. Since time delays are, in general, not integer multiples of the sampling periods, fitting parabolas to samples in a neighborhood of the cross correlation peak provides subsample accuracies. For noisy waveforms, estimators that rely on a single extremum of the correlation profile can have degraded performance. To reduce the effect of noise and increase accuracy, the algorithm in [1] matches multiple peaks of the cross correlation and autocorrelation for these waveforms. The algorithms in [2] use splines instead of parabolas. Other variations of this approach are described in [3] and the references therein. The algorithms in [4,5] also try to achieve subsample estimation accuracy. Approaches using the Hilbert Transform [6] and the state space realization method [7] have also appeared.

This paper builds on the time-delay estimation method described in [5]. This method uses the Discrete Fourier Transforms (DFTs) of multiple waveforms and achieves accurate subsample time delay estimates between these waveforms. It relies on applying linear phase shift operators to the DFTs of the waveforms to achieve least spectral differences between shifted DFTs in the frequency domain. The cost function is the sum of spectral differences between all pairs of shifted DFTs, and its minimization yields optimal time delays. However when a gradient based iterative algorithm is

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[1] İsmet Şahin acknowledges support under the National Institute for Standards and Technology American Recovery and Reinvestment Act Measurement Science and Engineering Fellowship Program Award 70NANB10H026 through the University of Maryland.

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http://dx.doi.org/10.1016/j.sigpro.2013.10.025

used to minimize the cost function, this approach becomes sensitive to the initial time-delay estimates. The global minimizer has a narrow basin of attraction and minimization algorithms often converge to a local minimizer when the initial estimates are not in the basin of the global minimizer. A new Hessian-based successive minimization approach with increased robustness is presented. An analytic expression for the Hessian matrix and a condition guaranteeing its positive definiteness are derived. This condition implies that narrower frequency bands, over which signals are aligned, yield larger basins of attraction. The successive minimization method uses this information to start with a cost function defined over a narrow frequency band and then increases the width gradually to generate successive cost functions each with smaller basins of attraction. We demonstrate that this new approach substantially increases robustness in simulated and real data. In the next section, we formulate the cost function and derive an analytic expression for its gradient.

2. Problem formulation

Consider a system of N frequency-band limited real valued signals $s_i(t) = s_0(t-\tau_i^o)+\eta_i(t)$ for $i = 1, 2, \ldots, N-1$, where τ_i^o represents the time delay between $s_i(t)$ and the reference signal $s_0(t)$, and $\eta_i(t)$ represents random noise on the ith signal. Let $y_i(l)$ for $i = 1, 2, \ldots, N-1$ denote the corresponding signals sampled at L equally spaced time samples $l = 0, 1, \ldots, L-1$ respectively. Let the L-point discrete Fourier transform (DFT) of $y_i(l)$ be given by the expression

$$Y_i(k) = Y_0(k)e^{-j\omega_k \tau_i^o} \quad \text{for} \quad i = 0, 1, 2, \ldots, N-1 \tag{1}$$

where $\tau_0^o = 0$ and $\omega_k = 2\pi k/L$. Consider the function

$$H(\underset{\sim}{\tau}) = \begin{bmatrix} G_{1,0} + \sum\limits_{n=1, n\neq 1}^{N-1} G_{1,n} & -G_{1,2} & \cdots & -G_{1,N-1} \\ -G_{2,1} & G_{2,0} + \sum\limits_{n=1, n\neq 2}^{N-1} G_{2,n} & \cdots & -G_{2,N-1} \\ \vdots & \vdots & & \vdots \\ -G_{N-1,1} & -G_{N-1,2} & \cdots & G_{N-1,0} + \sum\limits_{n=1, n\neq N-1}^{N-1} G_{(N-1),n} \end{bmatrix} \tag{6}$$

$$f(\tau_m, \tau_n) = \sum_{k=k_1}^{k_2} |Y_m(k)e^{j\omega_k \tau_m} - Y_n(k)e^{j\omega_k \tau_n}|^2 \tag{2}$$

defined over $0 \le k_1 < k_2 \le L-1$ with $\omega_{k_1} = 2\pi k_1/L$ and $\omega_{k_2} = 2\pi k_2/L$ representing the lower and upper frequency limits. Note that $f(\tau_m, \tau_n) \ge 0$ for $\tau_m \ne \tau_m^o$ and $\tau_n \ne \tau_n^o$. Rearranging terms in $f(\tau_m, \tau_n)$ yields

$$f(\tau_m, \tau_n) = C_f - 2\sum_{k=k_1}^{k_2} Re\{Y_m(k)e^{j\omega_k \tau_m} Y_n^*(k)e^{-j\omega_k \tau_n}\} \tag{3}$$

where $*$ denotes complex conjugate and $C_f = \sum_{k=k_1}^{k_2} \{|Y_m(k)|^2 + |Y_n(k)|^2\}$ is independent of τ_m and τ_n. Let $\underset{\sim}{\tau}^o = [\tau_1^o, \tau_2^o, \ldots, \tau_{N-1}^o]^T$ denote the vector of time delays. Following [5] we will estimate the time delay vector $\underset{\sim}{\tau}^o$ by minimizing the cost function:

$$J(\underset{\sim}{\tau}) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(\tau_m, \tau_n). \tag{4}$$

The function $J(\underset{\sim}{\tau})$ represents the total energy difference in the frequency domain between all possible shifted pairs of waveforms over the frequency range $[\omega_{k_1}, \omega_{k_2}]$. Clearly, the largest possible such range is when $k_1 = 0$ and $k_2 = L/2$ (which represents the ½ Nyquist frequency) and $J(\underset{\sim}{\tau})$ is minimized when $\underset{\sim}{\tau} = \underset{\sim}{\tau}^o$. Gradient based iterative algorithms such as steepest descent or conjugate gradient suggested in [5,8] require the evaluation of the gradient $\nabla J(\underset{\sim}{\tau})$ for minimizing $J(\underset{\sim}{\tau})$. The following expression for the pth entry in that vector was derived in [5]:

$$\frac{\partial J(\underset{\sim}{\tau})}{\partial \tau_p} = -4\sum_{k=k_1}^{k_2} Re\left\{ j\omega_k Y_p(k)e^{j\omega_k \tau_p} \sum_{\substack{n=0 \\ n\neq p}}^{N-1} Y_n^*(k)e^{-j\omega_k \tau_n} \right\} \tag{5}$$

for $p = 1, 2, \ldots, N-1$. An issue with minimizing $J(\underset{\sim}{\tau})$ using a gradient based algorithm is that initial estimates must be sufficiently close to $\underset{\sim}{\tau}^o$ to ensure convergence. The vector $\underset{\sim}{\tau}^o$ is not known a priori, therefore convergence of the algorithm is not certain. To reduce this uncertainty, a successive minimization method using the Hessian matrix of $J(\underset{\sim}{\tau})$ is proposed in the next section.

3. Successive minimization second order algorithm

The Hessian matrix $H(\underset{\sim}{\tau})$ of $J(\underset{\sim}{\tau})$ can be written:

where

$$G_{p,r} = G_{p,r}(\tau_p, \tau_r) = 4\sum_{k=k_1}^{k_2} \omega_k^2 Re\{Y_p(k)Y_r^*(k)e^{j\omega_k(\tau_p - \tau_r)}\} \quad \text{for} \quad p, r = 1, 2, \ldots, N-1 \quad \text{and} \quad p \ne r \tag{7}$$

and

$$G_{p,0} = G_{p,0}(\tau_p) = 4\sum_{k=k_1}^{k_2} \omega_k^2 Re\{Y_0(k)Y_p^*(k)e^{-j\omega_k \tau_p}\} \quad \text{for} \quad p = 1, 2, \ldots, N-1. \tag{8}$$

The sum of all entries in the pth row (or column) of $H(\underset{\sim}{\tau})$ is equal to $G_{p,0}$. The positive definiteness of $H(\underset{\sim}{\tau})$ is characterized in the following lemma:

Lemma. *If* $|\tau_p - \tau_p^o| < (L/8k_2)$ *and* $|\tau_r - \tau_r^o| < (L/8k_2)$, *then* $G_{p,r} > 0$ *and* $H(\underset{\sim}{\tau})$ *is positive definite.*