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Generalized Goertzel algorithm for computing the natural frequencies of cantilever beams

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ABSTRACT

The paper proposes a fast algorithm for accurate estimation of frequency within a specified, narrow range. The algorithm is useful for the identification of the natural frequencies of cantilever beams for damage detections purposes. The procedure is based on the generalized Goertzel algorithm combined with apriori knowledge of the natural frequencies intervals for cantilever beams given their physical characteristics. We compare our approach with the Chirp Z-transform and several frequency or time-frequency methods to illustrate its advantages for online damage detection.

1. Introduction

The detection of damages in cantilever beams using pre-computed natural frequencies is a challenging task approached by several authors using different techniques [1–4]. The main non-invasive techniques for computing the natural frequencies closely related to our approach are the spectral methods like the Goertzel algorithm [5,6], Chirp-Z transform [7,8] or time-frequency transforms like the short-time Fourier transform (STFT) [9], wavelets [4] etc.

The major drawback of such procedures is the fact that only tiny changes in the natural frequencies occur with the introduction of a damage, and also the technical difficulties encountered when computing these frequencies. Also for online (real-time) processing, the use of fast algorithms like the FFT and if possible even faster is almost compulsory.

With the recent introduction of the generalized Goertzel algorithm in [10], it became possible to exploit this algorithm in the search for the maximum peak in the intervals where the natural frequencies of the cantilever beams could be found.

The paper is structured as follows: In Section 2, we present the characteristics of the experiment i.e. the cantilever beam and the signal analyzed, supporting the practical motivation of our study. In Section 3, we overview the generalized Goertzel algorithm used for natural frequencies detection and compare it with Chirp-Z transform (CZT), its natural competitor. In Section 4, the proposed method for determining a set of ten natural frequencies is described, while in Section 5 we compare results of our method with time-frequency methods and we present in detail the advantages with respect to each of them. Lastly, the conclusions are drawn.

1.1. Notation

In the following text we assume a discrete signal x of length N, whose samples can be complex, $\{x[n]\} = \{x[0], x[1], \ldots, x[N-1]\}$. Symbol k can represent the

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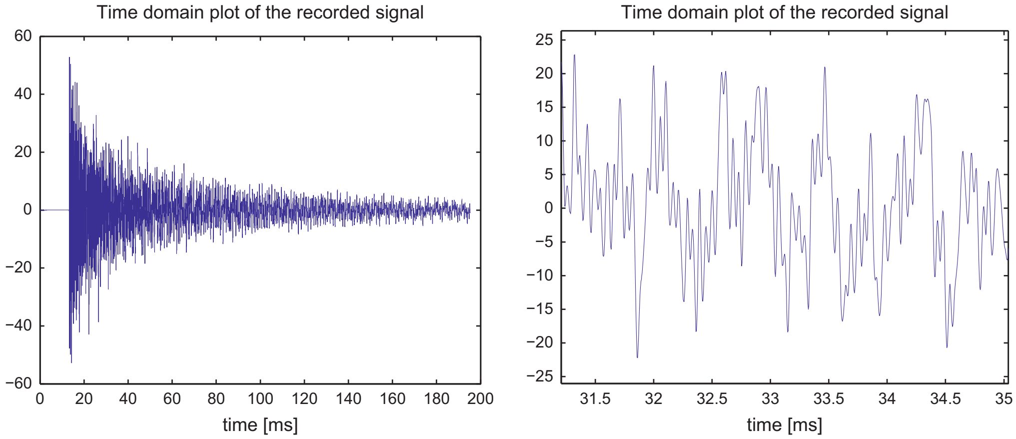

Fig. 1. Plot of the cantilever beam signal in the time domain, zoomed at the right.

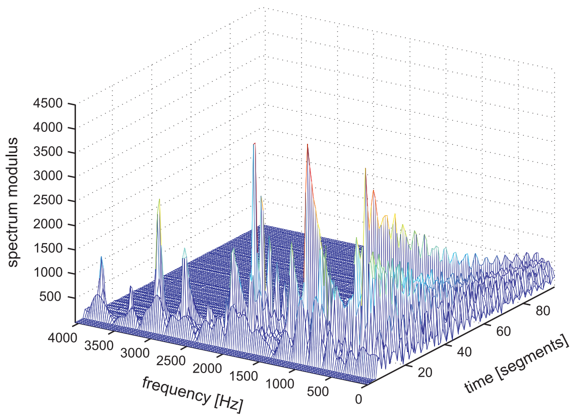

Fig. 2. Informative frequency plot over all time segments (FFT modulus). Rectangular window with 1024 samples and half-window overlap was used for this coarse step. Significant contributions of the natural frequencies are visible.

number (index) of the harmonic component in the DFT, thus $k \in \mathbb{N}$ as usual, however, in the framework of generalized Goertzel algorithm we allow working with $k \in \mathbb{R}$.

2. Natural frequencies of cantilever beams and identification of damages

Vibrations of the cantilever beam following a mechanical excitation are of (damped) harmonic type and therefore it is natural to try to accurately identify the frequencies contained in the oscillating waves.

A typical such signal (one of the signals we used for testing) in time domain is plotted in Fig. 1. For acquiring the signal, we placed an accelerometer on the free end of the unloaded beam. The sampling frequency was 26,500 Hz. The informative time-frequency contents of this signal is plotted in Fig. 2.

To develop an algorithm for detection and localization of damages it is necessary to have quantifiable indicators which characterize the dynamic behavior of the beam in the undamaged and the damaged state, respectively. One of the most used indicators in the non-invasive damage detection is the change in natural frequency occurring with the alteration of the beam geometrical and mechanical characteristics.

For our measurements and tests we used a steel cantilever beam having the following geometrical characteristics: length $l = 1000$ mm, width $b = 50$ mm, height $h = 5$ mm and consequently, for the undamaged state the cross-section $A = 250 \times 10^{-6}\ \text{m}^2$, moment of inertia $I = 520.833 \times 10^{-12}\ \text{m}^4$. The mechanical characteristics of the beam are mass density $\rho = 7850\ \text{kg/m}^3$, Young's modulus $E = 2.0 \times 10^{11}\ \text{N/m}^2$ and Poisson's ratio $\mu = 0.3$.

The damage in our case was simulated using the Finite Element Method (FEM) in 1000 points on the beam. Also real damages for direct measurements purposes were created on the beam in several most exposed points. The problem to highlight the appearance of a damage in beams using the natural frequency of a beam depends on the forces acting on it, as well as on the dimension of the damage (the cross-section reduction).

The described beam is considered as a reference. For beams with other dimensions (l, b, h) or mechanical characteristics (ρ, E, μ) the problem can be solved in a similar way by considering the scale influence.

During the numerical experiments, we aim at computing the first ten natural frequencies since this set is usually enough for the requirements of damage detection procedures like [1].

3. Goertzel algorithm and Chirp-Z Transform

From the problem description given above, it is clear that detecting tiny changes in the natural frequencies requires a procedure able to compute the signal's frequency spectrum with very fine resolution. This section is devoted to two algorithms, candidates for this purpose. The algorithms are described and compared here and a semiconclusion is drawn. All comparisons are done with the assumption that everything what allows precomputation is precomputed.

The FFT in its basic form is not included in this selection since it computes only DFT bins which are too coarse to be useful in our problem, and the number of samples which