Signal Processing 93 (2013) 1794–1803

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Robust H_∞ filtering for networked stochastic systems with randomly occurring sensor nonlinearities and packet dropouts ☆

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ARTICLE INFO

Article history:
Received 3 August 2012
Received in revised form
28 December 2012
Accepted 30 December 2012
Available online 11 January 2013

Keywords:
Robust H_∞ filtering
Networked stochastic systems
Markov chain
Sensor nonlinearity
Packet dropout

ABSTRACT

In this paper, the robust H_∞ filtering problem is investigated for networked stochastic systems with norm bounded uncertainties and imperfect multiple transmitted measurements. The considered imperfect measurements contain randomly occurring sensor nonlinearities and packet dropouts, which are represented by multiple independent Markov chains with partially unknown transition probabilities. A one to one mapping is constructed to map the multiple independent Markov chains to an augmented one for facilitating the resultant system analysis. A sufficient condition is established to guarantee the exponential mean-square stability with fast decay rate and a certain H_∞ performance level of the filtering error systems. Then, the parameters of the full-order filter are expressed in terms of linear matrix inequalities (LMIs). Finally, a numerical example is shown to demonstrate the effectiveness of the proposed method.

1. Introduction

The H_∞ filtering problem has been received persistent attention in the past half century [1]. Due to that, on the one hand, it does not require to know the precise system model and a prior information of the external noise compared with Kalman filter. On the other hand, the parameters of the H_∞ filter can be straightforwardly solved by the Riccati equation or the LMIs [2,3]. In addition, many researches are focused on stochastic systems, since many randomly occurred phenomena can be described as the stochastic processes [4–7]. Therefore, filtering for stochastic systems has become an active research field in the past two decades [8,9]. In [3], modified Riccati inequality was presented to solve the H_∞ filtering problem and the H_∞ performance was guaranteed based on the bounded real lemma. Full and reduced-order H_∞ filtering for stochastic linear system were studied using the LMI method [10,11], respectively. In [12,13], H_∞ filtering was extended to the fuzzy stochastic systems. However, little attention has been paid to robust H_∞ filtering for networked stochastic systems.

Another active research field in the past decade is the networked control systems (NCSs), which connect the systems with the controller or estimator through a communication channel [14]. NCSs become more significant for its convenience, such as shortening installation period, facilitating diagnosis and simplifying system maintenance [15]. The advantages of NCSs attract researchers to address its applications. In [16], the clustering based algorithm was used for the heterogeneous wireless sensor networks to improve the sensor network's lifetime. In [17], wireless sensor network based indoor positioning system was developed according to the rice distribution

☆ This work is supported by National Natural Science Foundation of PR China (NSFC: 61134007), National Natural Science Foundation of PR China (NSFC: 60736021), National Natural Science Foundation of PR China (NSFC: 61174029).

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http://dx.doi.org/10.1016/j.sigpro.2012.12.022

and the maximum likehood positioning algorithms. In [18], Markov jump system was introduced to analyze the stability of the teleopration system with transmission delay and packet dropouts. However, the capacity limit and the unreliable properties of the communication channel make the NCSs challenging. One of the interesting issue is how to compensate the packet dropouts caused by two factors [19–22]. One is the node failure or the data collision, which leads to the direct packet dropouts; the other is the long delay, i.e. the data will be abandoned once the delay exceeds a fixed value. Stochastic processes are effective models to describe the packet dropouts and one of the stochastic models is Bernoulli process, which was first proposed in [23]. The Bernoulli process was applied in the imperfect measurement fields of covariance analysis for Kalman filter [24], performance analysis for the H_∞ filter and LQG control design for the unstable systems [25]. To capture the possible temporal correlation of the channel variation, two states homogeneous Markov chain was proposed to model the packet dropouts and a number of significant works have been derived. In [26], a sufficient bounded condition of the estimation error covariance was achieved by introducing the peak covariance. By exploiting the system structure, the necessary and sufficient bounded condition of the Markov based packet dropouts was derived for some special structure systems [27]. A full-order H_∞ filter was designed for the NCS with Markov packet dropouts, and a necessary and sufficient condition was constructed to guarantee the exponential mean-square stability with H_∞ performance of the filtering error systems [28]. What we have to point out is that how to analyze the Markov chain based packet dropouts is still an interesting and challenge issue for the stochastic systems [29].

Physical sensors, as an essential part of the NCSs, inevitably show nonlinear characteristic in one or another form with environment changing [30]. Neglecting this nonlinear phenomenon will degrade the system performance or even lead the system to be unstable, which draws a great number of attentions from the researchers. According to LMI techniques, the H_∞ output feedback controller was designed in [31] and H_∞ filtering with sensor nonlinearity was analyzed in [12]. It is well known that sensors always distribute in a large area and work under various conditions. Therefore, the multiple communication channels with different packet dropout rates can better represent the actual NCSs [32]. Optimal estimator and robust filter for the multiple communication channels were considered in [33,34], respectively. In [35], feedback control over multiple fading channels was analyzed and the capacity of each channel was determined. However, it is still a challenging issue to study the influences simultaneously from unreliable communication channels and sensor nonlinearities.

Motivated by the above discussion, this paper studies the robust H_∞ filtering problem for networked stochastic systems subject to norm bounded uncertainties and packet dropouts. The considered imperfect measurements contain randomly occurring sensor nonlinearities and packet dropouts, which are modeled by m independent Markov chains with partially unknown transition probabilities. The main objective of this paper is to design a full-order filter such that the filtering error system is exponentially mean-square stable with a decay rate τ^{-1} and an H_∞ performance level γ. The solvability condition of the full-order filter is then expressed in terms of LMIs. Finally, a numerical example is shown to demonstrate the effectiveness of the proposed method. The main contributions of the paper can be summarized as follows: (1) A novel imperfect measurement model is proposed, where sensor nonlinearities and packet dropouts are contained, which can better reflect the actual conditions; (2) m independent Markov chains with partially unknown probabilities are introduced to describe the random cases of each measurement, which can deal with some harsh conditions that the information of the transition probabilities are hard to be obtained; (3) a one to one mapping is proposed to map the m independent Markov chains to an augmented one for facilitating the resultant system analysis.

This paper is organized as follows: The model of the filtering error system with imperfect measurements is proposed in Section 2, where the sensor nonlinearities and packet dropouts are described in a unified form via Markov chains, and an augmented Markov chain is presented based on a one to one map. In Section 3, the exponential mean-square stability with a decay rate τ^{-1} and an H_∞ performance level γ is analyzed. Then, the parameters of the filter are solved by LMIs methods in Section 4. A numerical example is presented in Section 5. Section 6 draws the conclusions and proposes the future work.

Notations: Throughout this paper, $\mathbb{R}$ stands for real numbers and $\mathbb{N}$ for nonnegative integers. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript M^T denotes the transposition of M. $\rho_{min}(M)$ and $\rho_{max}(M)$ stand for the minimum and the maximum eigenvalues of matrix M, respectively. $diag[\cdot]$ stands for a diagonal matrix. For the vector $\mathbf{x}\in\mathbb{R}^n$, $\|\mathbf{x}\|^2 \triangleq \mathbf{x}^T\mathbf{x}$. $\mathbb{P}(\mathbf{x}(t))$ denotes the probability of $\mathbf{x}(t)$. For notation $(\Omega,\mathcal{F},\mathcal{P})$, Ω stands for the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}(\mathbf{x}(t))$ defines the expectation of $\mathbf{x}(t)$. The notation $X\ge Y$ $(X>Y)$, where X and Y are symmetric matrices, means that $X-Y$ is positive semidefinite (positive definite). $l_2[0,\infty)$ is the space of square summable infinite sequence. $*$ denotes the term that is induced by symmetry.

2. Problem formulation

2.1. System description

Consider the following discrete-time linear system:

$$\begin{cases} \mathbf{x}(k+1)=(A+\Delta A(k))\mathbf{x}(k)+(B+\Delta B(k))\boldsymbol{v}(k) \\ \qquad\qquad +[(E+\Delta E(k))\mathbf{x}(k)+(G+\Delta G(k))\boldsymbol{v}(k)]\omega(k), \\ \mathbf{z}(k)=L\mathbf{x}(k), \quad k\in\mathbb{N} \end{cases} \tag{1}$$

and m measurements with randomly occurring sensor nonlinearities and packet dropouts are described as follows:

$$y_i(k)=\alpha_i(k)\phi_i(C_i\mathbf{x}(k))+\beta_i(k)C_i\mathbf{x}(k)+D_i\boldsymbol{v}(k), \tag{2}$$

where $\mathbf{x}(k)\in\mathbb{R}^n$, $y_i(k)\in\mathbb{R}$, and $\mathbf{z}(k)\in\mathbb{R}^q$ are the state, measurement received from sensor i and the signal to be