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Polynomial windows with low sidelobes' level

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Abstract

Some of the main advantages of polynomial windows are their low computational complexity and ability to easily change their frequency response modifying the values of their coefficients in the time domain. Kulkarni [Polynomial windows with fast decaying sidelobes for narrow-band signals, Signal Processing 83 (2003) 1145–1149] presented the coefficients obtained for such windows with fastest possible decaying sidelobes but their important limitation is very high level of the first sidelobe especially for windows with narrow mainlobe. In the article the results of the highest sidelobe level optimization for the family of polynomial windows are presented. Frequency characteristics of obtained windows are also compared to some well known ones such as Hann, Hamming, Blackman [On the use of windows for harmonic analysis with discrete Fourier transform, Proc. IEEE 66 (1) (1978) 51–83] and Nuttall [Some windows with very good sidelobe behavior, IEEE Trans. Acoust. Speech Signal Process. ASSP-29 (1) (1981) 84–91] windows.

Keywords: Polynomial window; Peak sidelobe; Mainlobe width; Time windows

1. Introduction

Typical windows used in signal processing applications are based on cosine-series such as Hann, Hamming or Blackman ones [1]. Apart from them many other functions have been proposed and optimized using various criterions i.e. Dolph–Chebyshev window minimizing the sidelobes' level without decay. Because of very high computational cost it is usually approximated by Hamming window based on cosine function expanded into series in practical DSP applications. Using Hann window the fastest decaying sidelobes for cosine-based windows family can be reached. In order to prevent the necessity of expanding cosine functions into series the polynomial windows with even better frequency domain properties can be used instead. In the paper [2] the coefficients for polynomial windows with fast decaying sidelobes have been presented. Although their computational complexity is very low, their main disadvantage is high level of the first sidelobe.

2. Frequency response of polynomial window

The definition of the polynomial window of order $2 \cdot N$ with only even exponents considered in the time interval from $-T/2$ to $T/2$ only is given by

$$w(t) = 1 + \sum_{n=1}^{N} C_{2n} \left(\frac{t}{T} \right)^{2n}. \tag{1}$$

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Frequency characteristics of such window can be determined using Fourier transform and expressed as [2]

$$W(f) = I_0 + \sum_{n=1}^{N} C_{2n} I_{2n}, \tag{2}$$

where I_{2n} is given in the recurrent form

$$I_0 = \frac{\sin(\pi f T)}{\pi f}, \tag{3}$$

$$I_{2n} = \frac{T[(\pi f T)\sin(\pi f T) + (2n)\cos(\pi f T)]}{4^n (\pi f T)^2} - \frac{n(2n-1)}{2(\pi f T)^2} \cdot I_{2n-2}. \tag{4}$$

In order to calculate the frequency characteristics and the sidelobes' levels properly an additional normalizing factor should be used guarantying that $W(0) = 1$ (0 dB). It can be expressed as

$$W(0) = 1 + \sum_{n=1}^{N} C_{2n} K_{2n}, \tag{5}$$

where

$$K_{2n} = \lim_{f \to 0} I_{2n}. \tag{6}$$

Multiplying both sides of Eq. (4) by $(\pi f T)^2$ the following expression for $f = 0$ and $T = 1$ is obtained

$$\frac{2n}{4^n} - \frac{n(2n-1)}{2} \cdot K_{2n-2} = 0 \tag{7}$$

so finally the expression (5) can be rewritten as

$$W(0) = 1 + \sum_{n=1}^{N} \frac{C_{2n}}{4^n (2n+1)}. \tag{8}$$

3. Proposed approach

An efficient solution decreasing the level of all sidelobes for a given window's order is resigning from forcing window's function and its derivatives to zero at $t = \pm T/2$ in the time domain. Setting $w(\pm T/2) = 0$ ensures 12 dB/octave asymptotic decay of sidelobes and N derivatives equal to 0 give $6(N+2)$ dB/octave. It leads to the family of polynomial windows of given order having the lowest sidelobe level with required asymptotic decay.

Increasing the order of polynomial serious reduction of the highest sidelobe level (HSLL) can be obtained and its cost is wider main lobe and 6 dB/octave less asymptotic decay in comparison to corresponding windows presented by Kulkarni. The increase of the width of main lobe (WML) is limited by the order of the polynomial and all eighth order windows have its main lobe's width less than 3 (i.e. triple as wide as for rectangular window) as shown in Table 1.

In practical applications the most popular windows are those having WML = 2 (Hann and Hamming windows) or WML = 3 (Blackman, Blackman-Harris or 3-term Nuttall windows [3])

Table 1
Parameters of polynomial windows with optimized highest sidelobe level (HSLL)

Window type	HSLL (dB)	WML (1/T)	Polynomial coefficients				
			C_2	C_4	C_6	C_8	C_{10}
$w_{0,2}$ [a]	−21.29	1.4303	−4	–	–	–	–
$w_{0,4}$	−39.48	1.9421	−8.2445	18.984	–	–	–
$w_{0,6}$	−48.58	2.3563	−11.6844	50.456	−78.855	–	–
$w_{0,8}$	−62.79	2.7992	−14.5628	89.5751	−276.7821	353.9600	–
$w_{0,10}$	−71.85	3.0283	−16.2559	116.4718	−470.469	1071.089	−1037.302
$w_{1,4}$ [a]	−27.72	1.8346	−8	16	–	–	–
$w_{1,6}$	−48.58	2.3565	−11.691	50.513	−79.003	–	–
$w_{1,8}$	−55.81	2.7482	−15.010	92.183	−273.039	321.879	–
$w_{1,10}$	−70.52	2.9847	−16.084	113.9596	−457.8596	1049.839	−1073.507
$w_{2,6}$ [a]	−33.35	2.2243	−12	48	−64	–	–
$w_{2,8}$	−55.81	2.7459	−14.990	91.937	−271.983	320.293	–
$w_{2,10}$	−62.52	3.1270	−18.267	143.302	−605.988	1365.73	−1286.10
$w_{3,8}$ [a]	−38.48	2.6046	−16	96	−256	256	–
$w_{3,10}$	−62.52	3.1290	−18.28	143.52	−607.30	1369.25	−1289.71

[a]Windows presented in [2].