Signal Processing 93 (2013) 554–572

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

When Van Gogh meets Mandelbrot: Multifractal classification of painting's texture

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ARTICLE INFO

Article history:
Received 9 September 2011
Received in revised form
8 January 2012
Accepted 14 January 2012
Available online 25 January 2012

Keywords:
Image processing
Texture classification
Regularity
Multifractal analysis
Wavelet leaders
Paintings
Van Gogh
Forgery detection
Period dating

ABSTRACT

Recently, a growing interest has emerged for examining the potential of Image Processing tools to assist Art Investigation. Simultaneously, several research works showed the interest of using *multifractal analysis* for the description of homogeneous textures in images. In this context, the goal of the present contribution is to study the benefits of using the wavelet leader based multifractal formalism to characterize paintings. After a brief review of the underlying key theoretical concepts, methods and tools, two sets of digitized paintings are analyzed. The first one, the *Princeton Experiment*, consists of a set of seven paintings and their replicas, made by the same artist. It enables examination of the potential of multifractal analysis in forgery detection. The second one is composed of paintings by Van Gogh and contemporaries, made available by the Van Gogh and Kröller-Müller Museums (Netherlands) in the framework of the *Image processing for Art Investigation* research program. It enables us to show various differences in the regularity of textures of Van Gogh's paintings from different periods, or between Van Gogh's and contemporaries' paintings. These preliminary results plead for the constitution of interdisciplinary research teams consisting of experts in art, image processing, mathematics and computer sciences.

1. Introduction

1.1. Image processing for art investigation

The ever growing power of digital devices (faster processors, better computers, higher resolution scanners, larger storage facilities, etc.) naturally and unavoidably gave birth to the desire of using such tools for Art Investigation. Yet, it is only recently, at the turn of the 3rd millennium, that conditions were met to transform this desire into some form of reality. Various research groups started to apply standard image processing tools to digitized painting, to develop new procedures, or to customize existing ones to meet the specificities of such an application (cf. [19] for an example of early contribution, [17,20] for review notes, and [12,21,22,8] for presentations of state-of-the-art and/or joint recent research contributions). With the development of computer-assisted and statistical signal-image processing tools, it is not the aim of scientists to supplant art historians, but rather to provide them with additional attributes that can be extracted automatically using objective and reproducible criteria. This will allow progress by diversification of the tools at hand. For paintings, it may for instance help to assess quantitative measures related to stylometry, brushstrokes, texture, etc. (cf. e.g., [24,13], where digital texture and brushstroke features are used to characterize paintings of Van Gogh). This may contribute to the formulation of answers to questions, such as what period was a painting created, is a painting authentic or a forgery, and has it been correctly attributed to an artist.

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0165-1684/$ - see front matter © 2012 Published by Elsevier B.V.
doi:10.1016/j.sigpro.2012.01.016

1.2. Wavelet and fractal for image processing

Over the last 15 years, elaborating on multiresolution decomposition and filter banks, wavelet analysis has become one of the inescapable image processing tools. In essence, wavelet coefficients evaluate the content of an image at a given space position $\mathbf{x}=(x_1,x_2)$ and a given analysis scale a. Wavelet coefficients usually take large values when the corresponding wavelet is located on any of the contours of the image, while they fluctuate around small values when the wavelet is located inside smooth textures. For an introduction, review and examples, the reader is referred to e.g., [14]. The statistical properties of wavelet coefficients have already been successfully used in stylistic analysis of paintings and forgery detection, cf. e.g., [9,12,15].

Fractal geometry refers to an analysis paradigm that relies on the idea that the richest part of the information to be extracted from an image lies in the way the statistics of some space-scale dependent quantities vary as a function of the analysis scale a. In other words, instead of basing the analysis on the search of specific features of space-scales, it is preferred to postulate that all space-scales are jointly and equally important and that the key information lies in the mechanisms relating them to each other. This dependence is usually postulated in the form of power laws: a^ζ (with ζ referred to as the *scaling exponent*) which explains why *fractal* is also termed *scaling* or *scale invariance*. Wavelet analysis consists in decomposing an image on elementary shapes (the wavelet basis) which are all deduced from three fundamental functions, the *mother wavelets*, by translation and dilation, see Eq. (1). Scaling invariance properties of the image will imply power-law behaviors of the wavelet coefficients. Therefore, in essence, wavelets constitute a *natural* decomposition system for characterizing fractal properties of images. Fractal tools can be used both for the analysis of contours and textures. There is a rich literature discussing the relevance of fractal paradigms to analyze or model natural images, a recent and interesting review can be consulted in [4]. In the context of Art, it was used in [18] to characterize some of Jackson Pollock's masterpieces.

1.3. Goals, contributions and outline

Beyond *fractal* analysis, essentially aiming at characterizing how *irregular* an object is globally by means of a single scaling exponent, *multifractal* analysis consists of a signal/image processing tool that concentrates on describing the fluctuations along space of the local regularity of the object, which requires the use of whole collections of scaling exponents. While popular for the analysis of 1D signals, multifractal analysis remained rarely used in image processing applications for both theoretical and practical reasons (cf. a contrario [2]). However, this situation has recently been changed when it was shown that a theoretically sound and practically efficient formulation of multifractal analysis could be obtained on the basis of wavelet leaders, a simple construction elaborating on 2D discrete wavelet transform coefficients, cf. [10,11,28,30,1]. This wavelet leader multifractal analysis constitutes a powerful tool for the analysis of textures in images, as detailed theoretically in [30] and explored practically in [29].

The present contribution aims at exploring the potential of the wavelet leader multifractal analysis for art painting texture classification. First (cf. Section 2), the principles and practical procedures underlying the wavelet leader multifractal analysis will be presented in a manner geared towards practitioners (hence avoiding theoretical developments and proofs, for which the reader will be referred to earlier publications). These procedures will be illustrated on several paintings. Then (cf. Section 3), it will be shown when and how the wavelet leader multifractal analysis enables to discriminate between original paintings and replicas. This will be embedded in the context of an original experiment conducted by the *Machine Learning and Image Processing for Art Investigation Research Group* at Princeton University (cf. www.math.princeton.edu/ipai/index.html). Finally (cf. Section 4), the wavelet leader multifractal analysis will be applied to a set of Van Gogh's and contemporaries' paintings, made available by the Van Gogh and Kröller-Müller Museums (The Netherlands) within the *Image Processing for Art Investigation* research project (cf. www.digitalpaintinganalysis.org/).

2. Multifractal analysis

2.1. Wavelet coefficients and global regularity

2.1.1. 2D discrete wavelet transform

An orthonormal wavelet basis in two dimensions is constructed from three smooth, compactly supported functions $\psi^{(1)},\psi^{(2)},\psi^{(3)}$, which are chosen such that the system

$$\psi^{(m)}_{j,(k_1,k_2)}(x_1,x_2)=2^{-j}\psi^{(m)}(2^{-j}x_1-k_1,2^{-j}x_2-k_2),\quad j,k_1,k_2\in\mathbb{Z},\ m=1,2,3 \tag{1}$$

constitutes an orthonormal basis of $L^2(\mathbb{R}^2)$. This system is called a wavelet basis, and the three functions $\psi^{(1)},\psi^{(2)},\psi^{(3)}$ its mother wavelets. Let $X(\mathbf{x})$ (with $\mathbf{x}=(x_1,x_2)$) denote a gray level image. We denote by $D_X^{(m)}(j,\mathbf{k})$ (with $\mathbf{k}=(k_1,k_2)$, $m=1$, 2, 3) the coefficients of the image X on this wavelet basis, which are given by the inner product with the basis functions, $D_X^{(m)}(j,\mathbf{k})=\langle X|\psi^i_{j,k}\rangle$. Note that in practice these wavelet coefficients are not computed as integrals, but using the classical pyramidal recursive algorithm supplied by the *fast wavelet transform*. Qualitatively, the coefficient $D_X^{(m)}(j,\mathbf{k})$ measures the amount of energy of the image X that is contained, in the spatial neighborhood of width $\sim 2^j$ located at position $(2^jk_1,2^jk_2)$, in the frequency bands localized around $\pm 2^{-j}$. For an introduction to the 2D discrete wavelet transform (2D DWT), the reader is referred to e.g., [14].

In the present contribution, it has been chosen to work with mother wavelets obtained as tensor products of the minimal compact support Daubechies wavelet families, which are parametrized by their number of vanishing moments N_ψ [5]. It has been discussed elsewhere that this family has ideal theoretical and practical properties with respect to scaling and fractal analysis (cf. e.g., [26]).