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PARAFAC-based unified tensor modeling for wireless communication systems with application to blind multiuser equalization

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Abstract

In some antenna array-based wireless communication systems the received signal is multidimensional and can be treated as a tensor (3D array) instead of a matrix (2D array). In this paper, we make use of a generalized tensor decomposition known as constrained Block-PARAFAC and propose a tensor (3D) model for the signal received by three types of wireless communication systems. The considered wireless communication systems are multiuser systems subject to frequency-selective multipath and employing multiple receiver antennas together with (i) oversampling or (ii) direct-sequence spreading or (iii) multicarrier modulation. The proposed modeling approach aims at unifying the received signal model of these systems into a single PARAFAC model. We show that the proposed model has a constrained structure, where model constraints and associated dimensions depend on each particular system. The proposed constrained Block-PARAFAC model is demonstrated by expanding the tensor using Kronecker products of canonical vectors. As an application of this model to tensor signal processing, a new tensor-based receiver is proposed for blind multiuser equalization, which combines PARAFAC-based modeling with a subspace method. Simulation results are presented to illustrate the performance of the proposed blind receiver.

Keywords: Alternating least squares; Antenna arrays; Blind equalization; Direct-sequence spreading; Frequency-selective multipath; Multicarrier modulation; Oversampling; Parallel factor analysis; Subspace; Tensor modeling; Wireless communications

1. Introduction

Most of existing array signal processing approaches rely on matrix (2D arrays) models for the received signal. In wireless communication systems, array signal processing is generally used at the receiver to mitigate multiuser (co-channel) interference, inter-symbol interference as well as to benefit from spatial diversity available in the wireless channel. Usually considered signal processing dimensions are *space* and *time* dimensions. In space–time matrix models, space dimension usually varies along the rows of the received signal matrix while time dimension varies along the columns.

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However, the main limitation of working with a matrix model for the received signal is its lack of inherent uniqueness. Regarding the blind recovery of information, blind algorithms generally take special (problem-specific) structural properties of the transmitted signals into account such as orthogonality, finite-alphabet, constant-modulus or cyclostationarity in order to overcome the non-uniqueness of matrix decompositions and successfully perform multiuser signal separation and equalization [1–3].

Unlike 2D (matrix) models, the use of 3D (tensor) received signal models in array signal processing problems result from the incorporation of a third "axis", also called dimension or mode, in addition to the usually considered *space* and *time* dimensions. For example, when temporal oversampling is used at the antenna array receiver, *oversampling* can be interpreted as the third dimension of the received signal. In a direct-sequence code division multiple access (DS/CDMA) system, *spreading* is the third dimension while in an orthogonal frequency division multiplexing (OFDM), *frequency* plays the role of this additional dimension. From a signal processing perspective, treating the received signal as a 3D tensor makes possible to simultaneously exploit the multiple forms of "diversity" inherent to it for a signal recovery purpose.

One of the most studied decompositions of 3D (or higher dimensional) tensors is called PARAFAC (parallel factor) analysis. PARAFAC was independently developed by Caroll and Chang [4] and Harshman [5] as a data analysis tool in psychometrics. It has also been widely studied in the context of chemometrics [6]. Several contributions bringing PARAFAC to the context of wireless communications have been carried out by Sidiropoulos and his co-workers (see [7] and the several references therein). In [8], the PARAFAC model has first appeared as a generalization of the ESPRIT method [9] for high-resolution direction finding. It has also been applied to the problem of multiuser detection for DS/CDMA systems in several works [10–13]. In [14], a PARAFAC receiver was also proposed for blind channel estimation in OFDM systems.

In this work, we present a new PARAFAC model for the received signal that is valid for some multiuser wireless communication systems subject to frequency-selective multipath fading. The proposed model unifies the received signal model of three systems: (i) a temporally oversampled system, (ii) a DS/CDMA system [15] and (iii) an OFDM system [16]. For all these systems an antenna array is assumed at the receiver front-end. The proposed tensor model assumes specular multipath propagation, where each user in the system contributes with a finite number of multipaths to the received signal. We show that the proposed model is subject to structural constraints in some of its component matrices, and that the same "general" PARAFAC model is shared by the three considered systems. For each particular system, the model can be obtained from the general model by making appropriate choices in the structure/dimension of its matrix components. For the DS/CDMA system, our tensor modeling approach generalizes those of [11,12], which consider a special propagation model with a single path per user. It also covers the tensor model proposed in [17] as a special case, which assumes multiple paths per user but does not consider a frequency-selective channel model.

This paper is summarized as follows. In Section 2, the standard PARAFAC decomposition is briefly introduced. The proposed tensor decomposition is also described in this section. In Section 3, the general system model and assumptions are presented. In Section 4, the proposed tensor decomposition is used to model the received signal associated with temporally oversampled, DS/CDMA and OFDM systems. Both scalar (multi-indexed) and matrix-slice notations are used to model the received signal in this section. In Section 5, we formalize the unified PARAFAC-based model for temporally oversampled, DS/CDMA and OFDM systems. An application of the PARAFAC model to blind multiuser equalization is presented in Section 6, and a new blind receiver algorithm that combines PARAFAC-based modeling with a subspace method is proposed. In Section 7, simulation results are presented to illustrate the performance of this PARAFAC-based receiver for different propagation scenarios. Finally, Section 8 concludes this paper.

Notation and properties: Some notations and properties are now recalled. $\mathbf{A}^{\mathrm{T}}, \mathbf{A}^{-1}$ and $\mathbf{A}^{\dagger}$ stand for transpose, inverse and pseudo-inverse of $\mathbf{A}$, respectively. The operator $Diag(\mathbf{a})$ forms a diagonal matrix from its vector argument; $BlockDiag(\mathbf{A}_1 \cdots \mathbf{A}_N)$ constructs a block-diagonal matrix of N blocks from its argument matrices; $D_i(\mathbf{A})$ forms a diagonal matrix holding the ith row of $\mathbf{A}$ on its main diagonal; $vec(\cdot)$ stacks the columns