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Finite-time filtering for discrete-time linear impulsive systems

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ABSTRACT

This paper investigates the finite-time filtering problem for a class of discrete-time linear impulsive systems. Sufficient conditions are derived in terms of linear matrix inequalities, such that the filtering error system is finite-time stable and has a guaranteed prescribed noise attenuation level for all non-zero noises. A new H_∞ finite-time filter is designed for the addressed discrete-time linear impulsive systems. Finally, a numerical example is exploited to show the effectiveness of the technique proposed in this paper.

1. Introduction

Impulsive differential equations can be used to describe many practical systems which experience state jump abruptly at certain moments of time. For the basic theory of impulsive differential equations the reader can refer to [1]. Some significant results have been reported in the literature, see [2–4] and references therein. For instance, paper [2] dealt with impulsive control of discrete systems with time delay. The filtering problems for the discrete time piecewise impulsive system were investigated in [3]. Paper [4] focused on the problem of globally exponential synchronization of impulsive dynamical networks.

The state estimation problem has long been one of the key research topics in communication and control domain. The shortcomings of Wiener filter have been overcome by the appearance of Kalman filtering theory. Exogenous disturbance can be regarded as arbitrary signals with limited energy when the statistical properties of the system disturbance are difficult to determine. In this case, H_∞ norm can be used for the performance index of filter. H_∞ filter has extensive applications in signal processing, radar design, fault detection and so on, see [5–11] and references therein. Paper [5] was concerned with the H_∞ filtering design for a class of discrete-time singular Markovian jump systems with time-varying delay and partially unknown transition probabilities. By embedding the unscented transform technique into the extended H_∞ filter structure, the unscented H_∞ filtering was carried out for a class of non-linear discrete-time systems in [6]. The problem of H_∞ filtering for a class of stochastic Markovian jump systems with impulsive effects was discussed in [7]. A delay-dependent approach to robust H_∞ filtering was proposed for linear discrete-time uncertain systems with multiple delays in the state in [8]. Robust filter design for linear discrete-time impulsive systems with uncertainty under H_∞ performance was considered in [9]. An H_∞ piecewise filtering method for T–S fuzzy systems with time delays was presented based on a piecewise Lyapunov–Krasovskii functional in [10]. Paper [11] was concerned with the problem of robust H_∞ filtering for uncertain impulsive stochastic systems under sampled measurements.

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It is worth mentioned that finite-horizon filtering has received much research interests, see [12–15] and references therein. A finite-horizon robust minimax filtering problem for time-varying discrete-time stochastic uncertain systems was studied in [12]. The robust finite-horizon filtering problem for a class of discrete time-varying systems with missing measurements and norm-bounded parameter uncertainties was considered in [13]. A finite-horizon robust Kalman filtering approach for discrete time-varying uncertain systems with additive uncertain-covariance white noises was presented in [14]. A new robust H_∞ filtering technique was developed for the discrete time-varying stochastic systems with polytopic uncertainties, randomly occurred non-linearities as well as quantization effects in [15].

Finite-time stability has attracted more and more attention due to its extensive applications. A system is said to be finite-time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a fixed time interval. The study of finite-time stability has been developed intensively in the last few years, for instance, see [16–21] and references therein. The sufficient conditions of finite-time stability of the linear system via impulsive control at fixed times and variable times were obtained in [16]. The finite-time analysis and design problems for continuous-time, time-varying linear systems were dealt with in [17]. Some finite-time control problems for discrete-time, time varying linear systems were addressed in [18]. The finite-time stability of linear time-varying singular systems with impulses was studied in [19]. Finite-time control problems for linear systems subject to time-varying parametric uncertainties and to exogenous constant disturbances were considered in [20]. Finite-time stabilization via dynamic output feedback of continuous-time linear systems was investigated in [21].

Since the increasing attention to the finite-time stability problem and the wide applications of H_∞ filter, we are motivated to consider the H_∞ finite-time filtering problem for discrete-time linear impulsive systems with disturbances. The finite-time filtering problem is different from the finite-horizon filtering problem. The finite-time stability of the system is investigated in the finite-time filtering problem besides a filter is designed for the system such that the estimation error output satisfies the H_∞ performance constraint for the given disturbance attenuation level. To the best of our knowledge, there are few results for finite-time filtering problem except [22], which is concerned with the problem of robust finite-time filtering for a class of non-linear Markov jump systems. The definition of finite-time stability in this paper is more general than Definition 1 in [22]. And the systems involved in this paper are definitely different from the one in [22].

In this paper, we firstly introduce a concept of finite-time stability for discrete-time linear impulsive systems and define the finite-time filtering problem for the proposed systems. Secondly, sufficient conditions for the filtering error system to be finite-time stable with noise attenuation are obtained. Thirdly, a design technique for the finite-time filter is given by way of solving a set of linear matrix inequalities. At last, a simulation example is used to illustrate the availability of the results.

2. Main results

Let R^n denote n-dimensional Euclidean space. $\mathbb{Z}^+$ denotes the set of positive integers. $l_2[0,T]$ is the space of square summable vector sequences over $[0,T]$, $T \in \mathbb{Z}^+$. I denotes the identity matrix of compatible dimension. For real symmetric matrices X and Y, the notation $X<Y$ means that the matrix $Y-X$ is positive definite. $\lambda_{max}(\cdot)$ represents the maximum eigenvalue of a symmetric matrix. The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry.

We consider the following discrete-time linear impulsive systems defined on $k \in [0,T]$:

$$x(k+1)=Ax(k)+Bw(k), \quad k \neq \tau_j, \tag{1a}$$

$$x(\tau_j+1)=Mx(\tau_j), \quad j \in \mathbb{Z}^+, \tag{1b}$$

$$y(k)=Cx(k)+D\nu(k), \tag{1c}$$

$$z(k)=Lx(k), \tag{1d}$$

$$x(0)=x_0, \tag{1e}$$

where $k \in \mathbb{Z}^+$, $x(k) \in R^n$ is the state vector, $y(k) \in R^l$ is the measured output vector, $z(k) \in R^p$ is the signal to be estimated. $w(k) \in R^q$ is the process noise and $\nu(k) \in R^r$ is the measurement noise, which both belong to $l_2[0,T]$. The time sequence $\{\tau_j\}$ are impulsive instants, which means those moments when the system state experiences jump abruptly. A, B, M, C, D and L are known real constant matrices with appropriate dimensions.

To study system (1a)–(1e), we suppose the following assumptions hold:

(H1) $\lim_{j\to+\infty}\tau_j=+\infty$, and there exists $m \in \mathbb{Z}^+$, such that $0<\tau_1<\cdots<\tau_m \le T<\tau_{m+1}<\cdots$;

(H2) For any given positive number d, the noise signals $w(k)$ and $\nu(k)$ are time-varying and satisfy

$$\sum_{k=0}^{T} w^T(k)w(k)\,dt \le \frac{d}{2}, \quad \sum_{k=0}^{T} \nu^T(k)\nu(k)\,dt \le \frac{d}{2}. \tag{2}$$

Before considering the finite-time filtering problem for system (1a)–(1e), we introduce a new concept of finite-time stability for discrete-time linear impulsive systems, which was inspired by [16,20,21].

Definition 1. Given two positive scalars c_1, c_2, with $c_1 \le c_2$, a positive definite matrix R, a positive definite matrix-valued function $\Gamma(\cdot)$ defined over $[0,T]$, with $\Gamma(0)<R$, system (1a)–(1e) is said to be finite-time stable with respect to $(c_1,c_2,T,R,\Gamma(\cdot),d)$ if

$$x_0^T R x_0 \le c_1 \Rightarrow x^T(k)\Gamma(k)x(k)<c_2, \quad \forall k \in [0,T].$$

Remark 1. When $\Gamma(k)$ is a constant matrix, let $\Gamma(k)=\Gamma(0)=R$, $\forall k \in (0,T]$ and $c_1<c_2$, then the formula in Definition 1 can degenerate into $x_0^T R x_0 \le c_1 \Rightarrow x^T(k) Rx(k)<c_2$, which is defined in Definition 1 in [21]. So Definition 1 in this paper is more general. The condition