Signal Processing 92 (2012) 2172–2179

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Subspace approach for two-dimensional parameter estimation of multiple damped sinusoids

Frankie K.W. Chan, H.C. So, Weize Sun *

Department of Electronic Engineering, City University of Hong Kong, Hong Kong

ARTICLE INFO

Article history:
Received 30 March 2011
Received in revised form
5 February 2012
Accepted 7 February 2012
Available online 17 February 2012

Keywords:
Dominant singular vector
Linear prediction
Weighted least squares
Two-dimensional parameter estimation
Spectral analysis

ABSTRACT

In this paper, we tackle the two-dimensional (2-D) parameter estimation problem for a sum of $K \geq 2$ real/complex damped sinusoids in additive white Gaussian noise. According to the rank-property of the 2-D noise-free data matrix, the damping factor and frequency information is contained in the dominant left and right singular vectors. Using the sinusoidal linear prediction property of these vectors, the frequencies and damping factors of the first dimension are first estimated. The parameters of the second dimension are then computed such that frequency pairing is automatically achieved. Computer simulations are included to compare the proposed approach with several conventional 2-D estimators in terms of mean square error performance and computational complexity.

1. Introduction

The problem of parameter estimation for $K \geq 1$ 2-D noisy sinusoids has received a great deal of attention. It is because in many applications such as source localization [1,2], radar imaging [3], vibrational analysis of circularly shaped objects [4], nuclear magnetic resonance (NMR) spectroscopy [5], wireless communication channel estimation [6], the corresponding signals can be well described by the 2-D sinusoidal model.

2-D Fourier transform is the direct nonparametric approach to address 2-D spectral estimation. In spite of its computational attractiveness when using fast Fourier transform, it suffers from poor resolution in resolving closely spaced frequencies and high-sidelobe effects [7,8]. In order to achieve higher resolution, the parametric approach, which assumes that the signal satisfies a generating model with known functional form, is a standard choice. Well known 2-D parametric solutions include maximum likelihood (ML) method [9] and subspace-based estimators such as multiple signal classification (MUSIC) [2,3,5], matrix enhancement and matrix pencil (MEMP) [10] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [4,6,11–13]. In the presence of additive white Gaussian noise, the ML scheme [9], which corresponds to a multi-dimensional peak search, can produce optimum estimation performance, that is, its mean square error (MSE) attains Cramér–Rao lower bound (CRLB). Comparing with the ML estimator, the subspace methodology whose underlying principle is to separate the received data into signal and noise subspaces via eigenvalue decomposition (EVD) or singular value decomposition (SVD), is more computationally efficient at the expense of suboptimality. The MUSIC algorithm [3] requires to find the K peaks in a 2-D cost function constructed from the noise eigenvectors. By constructing a Hankel-block-Hankel matrix whose size is larger than that of the data matrix, the MEMP method [10] decomposes the 2-D estimation problem into two 1-D problems related to each dimension, where generalized EVD and 2-D frequency pairing are needed. The ESPRIT algorithm [11] is similar to [10] in the sense that

* Corresponding author. Tel.: +852 67398755.
E-mail address: proton198601@hotmail.com (W. Sun).

0165-1684/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2012.02.003

Table 1
List of symbols.

Symbol	Meaning
†	Pseudoinverse
vec	Vectorization operator
$\otimes$	Kronecker product
$\odot$	Hadamard product
$\circ$	Khatri–Rao product
$\mathbf{I}_i$	$i \times i$ identity matrix
$\mathbf{0}_{i\times j}$	$i \times j$ zero matrix
$\tilde{\mathbf{a}}$	Noise-free value of $\mathbf{a}$
$\hat{\mathbf{a}}$	Estimate of $\mathbf{a}$
$[\mathbf{a}]_i$	ith element of $\mathbf{a}$
$[\mathbf{A}]_{i,j}$	(i, j) entry of $\mathbf{A}$
diag($\mathbf{a}$)	Diagonal matrix with vector $\mathbf{a}$ as main diagonal
blkdiag($\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_n$)	Block diagonal matrix with its diagonal elements are square matrices of $\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_n$
Toeplitz($\mathbf{a},\mathbf{b}^T$)	Toeplitz matrix with first column $\mathbf{a}$ and first row $\mathbf{b}^T$
vec($\mathbf{A}$)	$[\mathbf{a}_1^T\ \mathbf{a}_2^T\ \cdots\ \mathbf{a}_K^T]^T$ where $\mathbf{A}=[\mathbf{a}_1\ \mathbf{a}_2\ \cdots\ \mathbf{a}_K]$

the Hankel-block-Hankel matrix is exploited but it provides auto-pairing of the 2-D frequencies by making use of joint diagonalization and can deal with damped sinusoids. While [4], which addresses X-texture modes, that is, real-valued 2-D sinusoids with damping only in one dimension, can be considered as a modification to [11] by applying partial forward–backward averaging.

In this paper, we contribute to accurate and fast 2-D parameter estimation of multiple complex/real damped sinusoids based on the subspace methodology. We refer our approach to as principal-singular-vector utilization for modal analysis (PUMA), meaning that the principal singular vectors of the data matrix are effectively exploited in the estimation process. This work is a follow-up of [14] where the PUMA algorithm for a single damped/undamped real/complex tone or $K=1$ is devised and analyzed. The key ideas in [14] are to make use of the rank-one property of the corresponding 2-D noise-free data matrix and find the damping factor as well as frequency parameters for each dimension from the left and right principal singular vectors in a separable manner. In this paper, we extend [14] to the general case of $K \geq 2$ which is not a straightforward task. In particular, the damping factors and frequencies in each dimension are not directly related to the K principal singular vectors and 2-D parameter pairing needs to be addressed.

The rest of the paper is organized as follows. The algorithm development for multiple damped cisoids is provided in Section 2. According to the rank-K property of the 2-D noise-free data matrix, the corresponding left and right dominant singular vectors are characterized by the damping factors and frequencies in the first and second dimension, respectively. Making use of the dominant singular vectors, the parameters of interest at one dimension will first be accurately estimated according to an iterative procedure which utilizes the sinusoidal linear prediction (LP) property and weighted least squares (WLS). The damping factors and frequencies in another dimension are solved via another similar iterative algorithm such that pairing of the 2-D parameters is automatically achieved. Estimation for real tones is addressed in Section 3. In Section 4, simulation results are included to evaluate the performance of the PUMA approach by comparing with the ML [9] and ESPRIT algorithms [4,11] as well as CRLB. Finally, conclusions are drawn in Section 5. A list of mathematical symbols that are used in the paper is given in Table 1.

2. Estimation for complex sinusoids

In this section, we first devise the PUMA algorithm for estimating the parameters of multiple damped cisoids in additive noise. The signal model is

$$r_{m,n} = s_{m,n} + q_{m,n}, \quad m=1,2,\ldots,M,\ n=1,2,\ldots,N \tag{1}$$

where

$$s_{m,n} = \sum_{k=1}^{K} \gamma_k \alpha_k^m \beta_k^n \exp\{j(\mu_k m + \nu_k n)\} \tag{2}$$

is the noise-free signal. The γ_k is the complex amplitude, $\mu_k \in (-\pi,\pi)$ and $\alpha_k > 0$ are the frequency and damping factor in the first dimension while $\nu_k \in (-\pi,\pi)$ and $\beta_k > 0$ are the corresponding parameters in the second dimension, of the kth cisoid. The number of damped cisoids, namely, $K \geq 2$, is assumed known. Here we consider that the frequencies are distinct for at least one dimension. Without loss of generality, we assume that $M \geq N > K$ and all frequencies in the first dimension are not identical, that is, $\mu_k \neq \mu_l$, $k \neq l$. The additive noises $\{q_{m,n}\}$ are uncorrelated zero-mean complex white Gaussian processes with unknown variances σ_q^2. The task is to find $\{\mu_k\}$, $\{\nu_k\}$, $\{\alpha_k\}$, $\{\beta_k\}$ and $\{\gamma_k\}$, from the MN samples of $\{r_{m,n}\}$.

We first express (1) and (2) in matrix form as

$$\mathbf{R} = \mathbf{S} + \mathbf{Q} \tag{3}$$

where $[\mathbf{R}]_{m,n} = r_{m,n}$, $[\mathbf{S}]_{m,n} = s_{m,n}$ and $[\mathbf{Q}]_{m,n} = q_{m,n}$. Considering $s_{m,n}$ as a sum of K components of $(\alpha_k \exp\{j\mu_k\})^m \gamma_k(\beta_k \exp\{j\nu_k\})^n$, $k=1,2,\ldots,K$, it is easy to see that $\mathbf{S}$ can be factorized as

$$\mathbf{S} = \mathbf{G}\boldsymbol{\Gamma}\mathbf{H}^T \tag{4}$$

where $\mathbf{G} = [\mathbf{g}_1\ \mathbf{g}_2\ \cdots\ \mathbf{g}_K]$, $\boldsymbol{\Gamma} = \mathrm{diag}(\gamma_1,\gamma_2,\ldots,\gamma_K)$, $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2\ \cdots\ \mathbf{h}_K]$, $\mathbf{g}_k = [a_k\ a_k^2\ \cdots a_k^M]^T$, $\mathbf{h}_k = [b_k\ b_k^2\ \cdots b_k^N]^T$,