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A new structure for the design of wideband variable fractional-order FIR differentiators

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ABSTRACT

In this paper, a new structure is proposed for the design of wideband variable fractional-order (VFO) FIR differentiators. Part of subfilters in the conventional structure are replaced by the cascade of a prefilter and the modified subfilters, such that the performance of the designed system can be improved as well as the computational cost can be reduced. Several examples will be presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Conventionally, the transfer function

$$H_c(z,p)=\sum_{n=0}^{N}\sum_{m=0}^{M}h(n,m)p^m z^{-n} \tag{1}$$

is used for the design of variable fractional-order (VFO) differentiators with the desired frequency response

$$H_d(\omega,p)=e^{-jI\omega}(j\omega)^p=e^{-jI\omega}\hat{H}_d(\omega,p),\quad \omega_1\le|\omega|\le\omega_2,\ \ p_1\le p\le p_2, \tag{2}$$

where I is a prescribed delay,

$$\hat{H}_d(\omega,p)=|\omega|^p\left[\cos\left(\frac{p\pi}{2}\right)+j\,\mathrm{sgn}(\omega)\sin\left(\frac{p\pi}{2}\right)\right], \tag{3}$$

and $\mathrm{sgn}(\cdot)$ is a sign function. To approximate the desired frequency response (2), the filter coefficients $h(n,m)$ in (1) can be divided into even part and odd part by

$$h(n,m)=h_e(n,m)+h_o(n,m), \tag{4}$$

where

$$h_e\left(\frac{N}{2}+n,m\right)=\frac{1}{2}\left[h\left(\frac{N}{2}+n,m\right)+h\left(\frac{N}{2}-n,m\right)\right],\quad -\frac{N}{2}\le n\le\frac{N}{2},\ \ 0\le m\le M, \tag{5a}$$

$$h_o\left(\frac{N}{2}+n,m\right)=\frac{1}{2}\left[h\left(\frac{N}{2}+n,m\right)-h\left(\frac{N}{2}-n,m\right)\right],\quad -\frac{N}{2}\le n\le\frac{N}{2},\ \ 0\le m\le M, \tag{5b}$$

for even N. And the case for odd N can also be extended in a similar manner, which will be shown in Section 3. Hence, by defining

$$\hat{G}_{em}(z)=\sum_{n=0}^{N}h_e(n,m)z^{-n},\quad 0\le m\le M, \tag{6a}$$

$$\hat{G}_{om}(z)=\sum_{n=0}^{N}h_o(n,m)z^{-n},\quad 0\le m\le M, \tag{6b}$$

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$I=N/2$ and (1) can be represented by

$$H_c(z,p)=\sum_{m=0}^{M}[\hat{G}_{em}(z)+\hat{G}_{om}(z)]p^m, \quad (7)$$

which can be implemented by the structure shown in Fig. 1(a).

The design of fractional-order differentiators is an important topic of fractional calculus [1–4] which are widely applied in electromagnetic theory, fluid flow, automatic control, electrical networks and signal processing [5–10]. Recently, several works have been proposed for the design of fractional-order differentiators [11–18] as well as VFO differentiators [19–21]. Among them, the latter belongs to a branch of variable digital filters [22–30] which are designed such that their frequency characteristics can be adjustable online without redesigning the system. In this paper, the design of VFO FIR differentiators will be investigated for improving their performance or reducing the system complexity.

It is noted that for even N the subfilters $\hat{G}_{em}(z)$ and $\hat{G}_{om}(z)$ in (6) are type I and type III linear phase FIR filters [31], respectively, and the magnitude response of the latter will reduce to zero at folding frequency, which results in larger amplitude distortion in higher frequency for the design of wideband VFO FIR differentiators. In this paper, the wideband differentiator is denoted for that with $\omega_2 \geq 0.9\pi$ and a new structure will be proposed for further improving its performance or reducing its computational cost.

This paper is organized as below. Following the derivation at the beginning of this section, each of the type III subfilters in (6b) are replaced by the cascade of a type II prefilter and a type IV subfilter, which is extended from [30]. The proposed structure will be presented in Section 2 and the design formulation will be developed by applying the technique of [32–34]. To illustrate the effectiveness of the proposed method, several examples are presented in Section 3, including comparisons with the conventional method [21]. Finally, the conclusions are given in Section 4.

2. Proposed structure and formulation for the design of wideband VFO FIR differentiators

In this paper, the applied transfer function is given by

$$H(z,p)=z^{-K}\sum_{m=0}^{M}G_{em}(z)p^m+D(z)\sum_{m=0}^{M}G_{om}(z)p^m, \quad (8)$$

and the structure is shown in Fig. 1(b), where $D(z)$ is a type II prefilter, $G_{em}(z)$ and $G_{om}(z)$ are type I and type IV subfilters, respectively, and their transfer functions are represented by

$$D(z)=\sum_{n=0}^{N_d}d(n)z^{-n}, \quad N_d: \text{odd}, \quad (9a)$$

$$G_{em}(z)=\sum_{n=0}^{N_e}g_e(n,m)z^{-n}, \quad N_e: \text{even}, \; 0\leq m\leq M, \quad (9b)$$

$$G_{om}(z)=\sum_{n=0}^{N_o}g_o(n,m)z^{-n}, \quad N_o: \text{odd}, \; 0\leq m\leq M. \quad (9c)$$

To equalize the overall delay in (8), the integer K must satisfy

$$K+\frac{N_e}{2}=\frac{N_d}{2}+\frac{N_o}{2}, \quad (10)$$

i.e.

$$K=\frac{1}{2}(N_d-N_e+N_o). \quad (11)$$

It can be observed from Fig. 1(b) that the objective for improving the performance or reducing the computational cost can be achieved by using higher-order prefilter $D(z)$ and lower-order subfilters $G_{om}(z)$ and $G_{em}(z)$ when the cutoff frequency ω_2 is desirable that $\omega_2 \geq 0.9\pi$ under the cost of larger delay.

The frequency responses of $D(z)$, $G_{em}(z)$ and $G_{om}(z)$ can be represented by

$$D(e^{j\omega})=e^{-j(N_d/2)\omega}\sum_{n=1}^{(N_d+1)/2}\hat{d}(n)\cos\left(\left(n-\frac{1}{2}\right)\omega\right), \quad (12a)$$

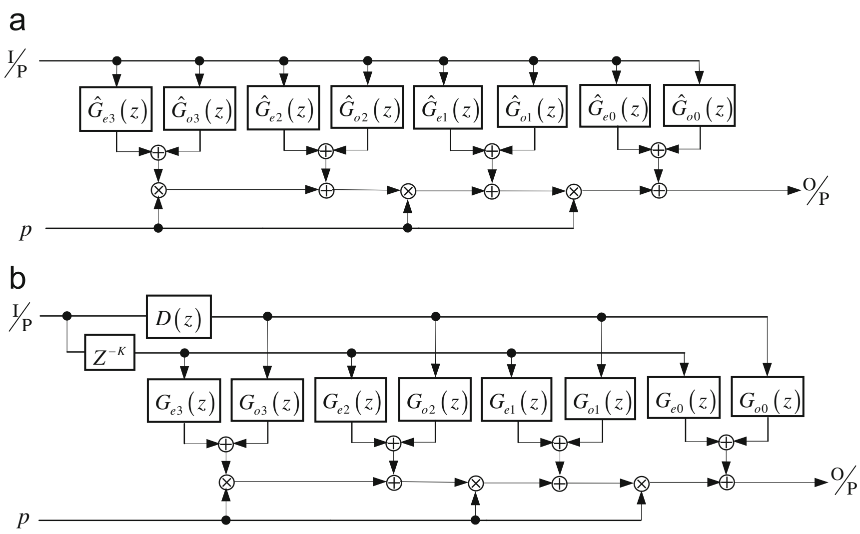

Fig. 1. Structures for the design of VFO FIR differentiators (M=3). (a) Conventional method [21]. (b) Proposed method.