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Minimum variance time of arrival estimation for positioning

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ABSTRACT

Positioning systems based on time of arrival (TOA) rely on an accurate estimation of the first signal arrival which is the only one bearing position information. In this context, first arrival path detectors based on the minimum variance (MV) and the normalized minimum variance (NMV) criteria are robust against multipath propagation and non-line-of-sight (NLOS) situations. The aim of the paper is twofold. On the one hand, efficient implementations of those criteria will be presented. On the other hand, two polynomial rooting procedures based on the MV criterion will be exposed. As it will be shown, they lead to better performance than the traditional MV versions based on a grid search. In general, these methods imply finding the roots of a polynomial with complex coefficients. In order to reduce their computational burden a conformal mapping is proposed herein.

1. Introduction

Location systems based on TOA require an accurate estimation of the first time-delay. Unfortunately, in wireless channels, multipath propagation and non-line-of-sight (NLOS) situations bias the estimation of the first signal arrival. In this context, detectors based on the maximum of the impulse response [1], despite their low computational burden, are not suitable because the delayed replicas displace the maximum of the response. Analogously, conventional correlators cannot separate the direct path from the echoes if they are close or the direct propagation path is highly attenuated [2]. On the other hand, conventional parametric methods based on the maximum likelihood (ML) criteria provide a complete channel description at expenses of a heavy computational burden, leading to an expensive multi-dimensional search (e. g. [3–5]).

As it is well-known, time delay estimation in the frequency domain and spectral (or spatial) analysis are closely connected [6]. In this context, high resolution spectral techniques such as multiple signal classification (MUSIC) and root-MUSIC can be used as TOA estimators [2,7–9]. These methods are commonly referred in the literature as subspace or singular value decomposition (SVD) methods and are based on the separation of the signal and the noise subspaces. This characterization is costly and needs the knowledge of the number of propagation paths. Another well-known SVD technique which avoids the traditional scanning of MUSIC is total least squares estimation of signal parameters via rotational invariance technique (TLS-ESPRIT) and it was applied to time delay estimation in [10]. Bearing in mind the analogy between time delay estimation and spectral analysis, high resolution methods based on the minimum variance (MV) [11] and on the normalized minimum

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variance (NMV) [12] criteria were successfully developed and applied to location systems in [13,14]. These techniques can provide an accurate estimation of the first signal arrival at an affordable cost in high multipath environments, even when the line of sight (LOS) signal is strongly attenuated. Herein, efficient implementations of the grid search of the MV and NMV TOA estimators based on fast Fourier transform (FFT) are described, allowing a significant reduction on the computational burden [22]. Furthermore, new polynomial approaches which transform the grid search of the MV into a polynomial rooting procedure are exposed in this paper. The first of these techniques is obtained deriving the MV power delay profile (PDP) and it is referred herein as root derivative minimum variance (RDMV). This method is closely related to the second polynomial algorithm, the root minimum variance (RMV) estimator which was briefly introduced in [15] and later on analyzed in detail in [16] in the context of angle of arrival (AOA) estimation. The performance of the RMV algorithm as a TOA estimator in a location system is explored in this paper yielding accurate results, even better than those of the NMV estimator, at least for moderate or low SNR. In order to reduce the computational cost of the rooting techniques a conformal transformation [17] is described herein. This mapping circumvents the complex nature of the polynomials transforming them into a polynomial with real coefficients. As efficient rooting procedures based on real arithmetic are applicable, a significant reduction on the computational burden can be achieved without accuracy reduction. A similar approach was applied to MUSIC in the context of AOA estimation in [18].

The main goal of this paper is to analyze the performance of several TOA estimators based on the minimum variance criterion. The integration of partial results presented by the authors in previous works in a general framework provides a more meaningful view of these techniques. To compare the performance of these techniques with a theoretical bound, the expression of the Cramér-Rao bound is derived herein. It is based on the frequency domain signal model of the received signal and it leads to a simple and compact expression.

The paper is organized as follows. In Section 2 the signal model is described. The different TOA algorithms are derived in Section 3. Performance analysis of these methods and simulation results in a realistic mobile scenario are shown in Section 4. Finally, concluding remarks are addressed in Section 5.

2. Signal model

Let us consider N observations of a received signal consisting of L superimposed delayed replicas of a known waveform $g(t)$ embedded in noise:

$$y(t;n)=\sum_{i=0}^{L-1}\alpha_i(n)g(t-\tau_i)+\eta(t;n)\quad n=1,\ldots,N \tag{1}$$

being $\alpha_i(n)$ and τ_i the complex time-varying amplitude and the time-delay of the i-th arrival, respectively. Without loss of generality let us assume $\tau_0<\tau_1<\cdots<\tau_{L-1}$. The additive white Gaussian noise is denoted by $\eta(t;n)$.

In the frequency domain the expression (1) is transformed into a weighted sum of complex exponentials embedded in noise:

$$Y(\omega;n)=\sum_{i=0}^{L-1}\alpha_i(n)G(\omega)e^{-j\omega\tau_i}+N(\omega;n) \tag{2}$$

where $Y(\omega;n)$ is the noisy frequency domain observed signal, $G(\omega)$ denotes the Fourier transform of $g(t)$ and $N(\omega;n)$ is the additive noise in the frequency domain. Sampling (2) at $\omega_m=\omega_0 m$ for $m=0,1,\ldots,M-1$ and $\omega_0=2\pi/M$ and rearranging the frequency domain samples $Y(m;n)$ into the vector $\mathbf{y}_n\in\mathbb{C}^{M\times 1}$ yields,

$$\mathbf{y}_n=\mathbf{GEh}(n)+\boldsymbol{\eta}(n) \tag{3}$$

where the matrix $\mathbf{G}\in\mathbb{C}^{M\times M}$ is a diagonal matrix whose components are the frequency samples $G(\omega_m)$ and the matrix $\mathbf{E}=[\mathbf{e}_{\tau_0}\ \mathbf{e}_{\tau_1}\ \cdots\ \mathbf{e}_{\tau_{L-1}}]\in\mathbb{C}^{M\times L}$ contains the delay-signature vectors associated to each arriving delayed signal, with $\mathbf{e}_{\tau_i}=[1\ \ e^{-j\omega_0\tau_i}\ \ \cdots\ \ e^{-j\omega_0(M-1)\tau_i}]^T$. The channel fading coefficients are arranged in the vector $\mathbf{h}(n)=[h_0\ \cdots\ h_{L-1}]^T\in\mathbb{C}^{L\times 1}$, and the noise samples in vector $\boldsymbol{\eta}(n)\in\mathbb{C}^{M\times 1}$.

3. MV TOA estimation

TOA can be estimated as the first delay that exceeds a given threshold in the power delay profile, defined as the signal distribution of the power with respect to propagation delays. The PDP can be obtained by estimating the power of the signal filtered at each delay [19], $P(\tau)=\mathbf{w}^H(\tau)\mathbf{R}_y\mathbf{w}(\tau)$, where $\mathbf{R}_y$ is the correlation matrix of the received signal estimated from a collection of N observations within the coherence time of the delays and $(\cdot)^H$denotes the conjugate transpose operator. The vector $\mathbf{w}(\tau)$ can be obtained by applying the minimum variance criterion which consists of the minimization of the output power with the constraint $\mathbf{w}(\tau)^H\mathbf{Ge}_\tau=1$, which guarantees that the signal is not distorted at the explored time delay. This constrained minimization yields to a closed-form for the MV PDP estimation:

$$P_{MV}(\tau)=\frac{1}{\mathbf{e}_\tau^H\mathbf{G}^H\mathbf{R}_y^{-1}\mathbf{Ge}_\tau} \tag{4}$$

In the traditional approach, the value of the PDP is evaluated in a grid of time delays. Nevertheless, as the authors described in [20,21], the expression (4) can be reformulated as a discrete Fourier transform (DFT) and efficiently implemented using a FFT of length P, where P is the number of grid points,

$$P_{MV}(\tau)=\frac{1}{\sum_{k=-M+1}^{M-1}D(k)e^{-j\omega_0\tau k}} \tag{5}$$

being $D(k)=\sum_{n=1}^{M-k}[\boldsymbol{\Psi}]_{n,n+k}$ where $[\boldsymbol{\Psi}]_{i,j}$ is the element corresponding to the i-th row and the j-th column of the matrix $\boldsymbol{\Psi}=\mathbf{G}^H\mathbf{R}_y^{-1}\mathbf{G}$. The inversion of the correlation matrix can be avoided using the Gohberg–Semencul formula [19], being the computational burden of the whole process $O(M^2+P\log_2 P)$.