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Generalization of order distribution concept use in the fractional order system identification

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ABSTRACT

In this paper, the order distribution concept in the frequency domain identification has been extended to include fractional order systems having poles and zeros simultaneously. The existing nonlinear optimization problem appeared when both poles and zeros, is are changed to a quadratic problem that can be solved using least squares algorithms. To collect the required data, system is excited by a multi sine input signal with appropriately selected frequencies. Then a nonparametric identification in frequency domain is accomplished to calculate the empirical transfer function estimate (ETFE). This estimate is then used to implement the frequency domain identification on all defined members of the model set to estimate the model parameters in noise free and disturbed cases.

1. Introduction

Fractional calculus as a mathematical topic was introduced by Leibniz and L'Hospital about 315 years ago. This mathematical representation has recently been used to describe phenomena observed in such processes as electro-magnetic [1], electro-mechanic [2], heat transfer [3,4], electro-chemical [5], visco-elastic materials [6] and biology [7–9]. Some basic concepts of fractional systems which constitute an important branch of the fractional calculus have been discussed in [10–13]. There are many researches conducted on fractional calculus applications in control [14–17], image processing [18,19], signal processing [20], and calculational mathematics [21]. Till now many different aspects of the fractional order system applications have been studied which includes stability analysis [22–24], system identification [25–35], system approximation [36], synchronization [37,38], dynamical behavior analysis [39–42], controller order reduction [43], evolutional optimization [44] and chaos [45].

System identification is an essential part of the control engineering and incorporates many different approaches and techniques to find a kind of relation among inputs and outputs of a system. The mentioned relations can be represented either in time domain or frequency domain and also can be determined either by time domain or frequency domain measurements from the system [46]. In [29,47], the frequency domain representation of a fractional order system is determined in the equation error (EE) and output error (OE) based model structures, respectively, using time domain measurements. In deterministic case of the frequency domain identification, the first work was reported in [48] where the existing nonlinear least squares problem is replaced by a linear least squares one by multiplying the equation error with the denominator of the transfer function. Authors in [49] overcame the lack of sensitivity to low frequency errors of the linear least squares estimator by an iterative

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procedure. In [50,51] authors have solved the existing nonlinear least squares problem using the Newton–Gauss iteration scheme for continuous and discrete time model, respectively. The identification of fractional models in a way rather close to [48], i.e. restricted to all poles commensurate transfer functions has been performed in [30,52] and the similar method for all poles complex commensurate order system has been reported in [53]. Some of the frequency domain identification methods for integer order models were extended to fractional ones in [54]. In [55], the coherence function has been used to design an appropriate frequency domain identification experiment. In [56] an algorithm is presented for identifying state-space models from the frequency-domain data.

In this paper a new method is proposed to identify fractional pole-zero transfer functions using the frequency domain data. A combinational procedure using complex curve fitting and nonlinear optimization is employed to obtain order distribution of the numerator and denominator of the transfer function. This can be considered as an extension to the work presented in [30] where only all-pole transfer functions were studied. Furthermore, a new approach is introduced to improve the condition number of the involved matrices in the calculations. The proposed method is examined first by identification of an example deterministic system and then is performed for a special stochastic system by pre-processing of the measured data.

The rest of the paper has been structured as follows. Section 2 includes materials related to the frequency domain identification of fractional order systems. Generalization of an existing method on all-pole transfer functions to pole-zero transfer functions is described and its associated problems are discussed. Some numerical examples are presented in Section 3 to explain how the new method works and what performance should one expect. Some validation results on the obtained models are given in Section 4 which is based on time domain responses. The paper is concluded in Section 5.

2. Using order distribution approach for frequency domain identification

In [30], an all-pole transfer function with the following structure was considered,

$$G(s) = \frac{1}{P(s)} = \frac{1}{\int_0^{q_{\max}} \bar{k}(q) s^q \, dq}. \quad (1)$$

Assuming that the order distribution, $k(q)$, is such that the integral converges, relation (1) can be approximated by the following one using an Euler approximation at each frequency,

$$\sum_{n=0}^{N} k_n (j\omega)^{nQ} Q \approx \frac{1}{G(j\omega)}, NQ = q_{\max}, \quad (2)$$

where Q is the sample width in the variable q. The following matrix form generalizes (2) for L different frequency response samples

$$Q \begin{bmatrix} 1 & (j\omega_1)^Q & \dots & (j\omega_1)^{NQ} \\ 1 & (j\omega_2)^Q & \dots & (j\omega_2)^{NQ} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (j\omega_L)^Q & \dots & (j\omega_L)^{NQ} \end{bmatrix} \begin{bmatrix} k_0 \\ k_1 \\ \vdots \\ k_N \end{bmatrix} \approx \begin{bmatrix} 1/G(j\omega_1) \\ 1/G(j\omega_2) \\ \vdots \\ 1/G(j\omega_L) \end{bmatrix}. \quad (3)$$

This equation is written in compact form as $QWk = g$ and has the following solution for unknown k provided that $2L \geq N+1$.

$$k = Q^{-1}(W^T W)^{-1} W^T g. \quad (4)$$

Two points should be considered here. First, in discrete order distributions, $\bar{k}(q)$ is Dirac-delta function and therefore there would be no need for approximation of the integral in (1) to get to (2). In other words, when one considers the discrete order distributions Q should not be appeared as a coefficient in (3). Second, k in (4) may get complex values, sometimes with big imaginary parts, due to the computer round off errors, whereas it must be real value.

To consider these two points in the discrete order distributions cases, one should remove coefficient Q in (3) and also use a complex curve fitting (such as one in [48]) to solve it for k, i.e.

$$k = [\mathrm{Re}(W^H W)]^{-1} \mathrm{Re}(W^H g). \quad (5)$$

The presented method can be generalized to include transfer functions with both zero(s) and pole(s). Allowing the restriction on the maximum possible numerator and denominator order, a general system representation becomes

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\int_{qm_0}^{qm_M} \bar{m}(q) s^q \, dq}{\int_{qn_0}^{qn_N} \bar{n}(q) s^q \, dq} = \frac{m_M s^{qm_M} + m_{M-1} s^{qm_{M-1}} + \dots + m_1 s^{qm_1} + m_0 s^{qm_0}}{s^{qn_N} + n_{N-1} s^{qn_{N-1}} + \dots + n_1 s^{qn_1} + n_0 s^{qn_0}}, \quad (6)$$

where qm_0, qn_0 are the lower limits and qm_M, qn_N are the upper limits on the differential orders. Notice that the denominator polynomial is monic and also qm_0, qn_0 are not nonzero simultaneously.

$\bar{m}(q)$ and $\bar{n}(q)$ are the order distribution of the numerator and denominator, respectively, (as it is shown in Fig. 1 for $\bar{m}(q)$). In discrete case, order distribution contains Dirac-delta functions at distinct orders of the $\bar{m}(q)$ or $\bar{n}(q)$.

Now we choose L frequency samples by replacing s by $j\omega$ where ω is linearly spaced in $[\omega_{\min}, \omega_{\max}]$.

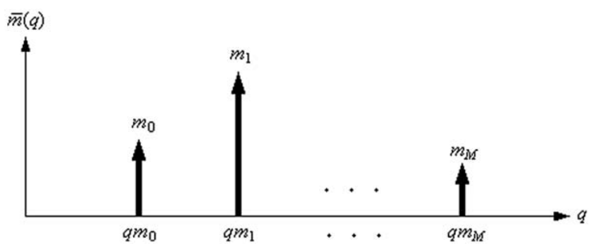

Fig. 1. Order distribution of numerator $\bar{m}(q)$.