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Stability analysis for 2-D systems with interval time-varying delays and saturation nonlinearities

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A R T I C L E I N F O

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A B S T R A C T

This paper addresses the delay-dependent stability problem for 2-D discrete-time systems described by the Fornasini–Marchesini second state-space model with interval time-varying delays and saturation nonlinearities. By using linear matrix inequalities (LMIs) method, the delay-range-dependent conditions are derived, which not only depend on the difference between the upper and lower delay bound but also on the upper delay bound of the interval time-varying delays. Finally, numerical example is given to illustrate the effectiveness of the proposed technique.

1. Introduction

During the past few decades, two-dimensional (2-D) systems have received intensive interest since it take an important role in both theory and application such as multi-dimensional digital filtering, linear image processing, signal processing, process control, and so on [1–3]. As is well known, the values of filter coefficients are stored in registers which have a finite wordlength in hardware implementations. When 2-D systems are implemented in the finite wordlength format, the saturation nonlinearities often occur in 2-D systems. However, such nonlinearities may lead to instability. As a result, the study of stability problem for 2-D systems with saturation nonlinearities is important not only for its theoretical interest but also for application to practical filter design. Recently, a great number of stability conditions for 2-D systems with saturation nonlinearities have been reported in the literature [4–14].

In 2-D systems, time delays must be taken into account due to the finite speed of information processing. Time delays may lead to oscillation, instability, and poor performance. Therefore, the study of 2-D systems with time delays has received much attention in recent years; see, for example [15–20], and the references therein. In practical implementations of 2-D systems, time delays and the saturation nonlinearities are frequently encountered. Such systems can be represented as 2-D systems with time delays and saturation nonlinearities. Recently, the delay-independent stability problem for one-dimensional (1-D) systems with constant time delays subject to saturation nonlinearities has been addressed [21,22]. However, to the author's knowledge, the delay-dependent stability problem for 2-D systems with both interval time-varying delays and saturation nonlinearities has not been fully investigated.

This paper is concerned with the asymptotic stability of 2-D systems described by the Fornasini–Marchesini second local state-space model under interval time-varying delays and saturation nonlinearities. Based on Lyapunov stability

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theory, the delay-range-dependent conditions are derived to ensure the asymptotic stability for the addressed system. Furthermore, the conditions are expressed in terms of the linear matrix inequalities [23], which not only depend on the difference between the upper and lower delay bound but also on the upper delay bound of the interval time-varying delays. An illustrative example is given to show the effectiveness and applicability.

The notation used throughout the paper is quite standard. $\mathbb{Z}$ is the set of nonnegative integers, $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. P^{T} stands for the transpose of a matrix P, and $P>0$ (<0) means that $P=P^{\mathrm{T}}$ is positive definite (negative definite). The boldface characters represent matrix variables. In block matrices, * is used as an ellipsis for the transpose of the block at the symmetric position.

2. Problem formulation

Consider the 2-D discrete-time systems with interval time-varying delays and saturation nonlinearities described by the Fornasini–Marchesini second state-space model [24]

$$x(i+1,j+1)=\phi(\xi(i,j))=[\phi_1(\xi_1(i,j))\ \phi_2(\xi_2(i,j))\ \cdots\ \phi_n(\xi_n(i,j))]^{\mathrm{T}},$$

$$\xi(i,j)=[\xi_1(i,j)\ \xi_2(i,j)\ \cdots\ \xi_n(i,j)]^{\mathrm{T}}=A\begin{bmatrix}x(i+1,j)\\ x(i,j+1)\end{bmatrix}+A_d\begin{bmatrix}x(i+1,j-d_1(j))\\ x(i-d_2(i),j+1)\end{bmatrix},\tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector. The matrices

$$A=[A_1\ A_2],\quad A_d=[A_{d_1}\ A_{d_2}],$$

where A_1, A_1, A_{d_1} and A_{d_2} are known constant matrices with compatible dimensions. $d_1(j)$ and $d_2(i)$ are time-varying delays along vertical and horizontal directions, respectively. We assume $d_1(j)$ and $d_2(i)$ satisfying

$$d_{1L}\le d_1(j)\le d_{1H},\quad d_{2L}\le d_2(i)\le d_{2H},\tag{2}$$

where d_{1L}, d_{1H}, d_{2L}, and d_{2H} are constant positive scalars representing the lower and upper delay bounds along vertical and horizontal directions, respectively. $\phi_k(\xi_k(i,j))$ is saturation nonlinearities given by

$$\phi_k(\xi_k(i,j))=\left\{\begin{array}{ll}1, & \xi_k(i,j)>1\\ \xi_k(i,j), & -1\le\xi_k(i,j)\le 1\\ -1, & \xi_k(i,j)<-1\end{array}\right\},\quad k=1,2,\ldots,n.\tag{3}$$

The boundary conditions associated with system (1) are defined as follows:

$$\begin{aligned}&x(i,j)=s_{ij},\quad \forall\, 0\le i<k_1,\ j=-d_{1H},-d_{1H}+1,\ldots,0,\\ &x(i,j)=0,\quad \forall\, i\ge k_1,\ j=-d_{1H},-d_{1H}+1,\ldots,0,\\ &x(i,j)=t_{ij},\quad \forall\, 0\le j<k_2,\ i=-d_{2H},-d_{2H}+1,\ldots,0,\\ &x(i,j)=0,\quad \forall\, j\ge k_2,\ i=-d_{2H},-d_{2H}+1,\ldots,0,\\ &s_{00}=t_{00},\end{aligned}\tag{4}$$

where $k_1<\infty$ and $k_2<\infty$ are positive integers, s_{ij} and t_{ij} are given vectors. The following definition and lemma will be used later.

Definition 1 (*Paszke et al.* [19]). The system (1) is asymptotically stable if $\lim_{\ell\to\infty}\chi_\ell=0$ for all bounded boundary conditions in (4), where

$$\chi_\ell=\sup\{\|x(i,j)\|: i+j=\ell, i,j\ge 1\}.$$

Definition 2 (*Gao et al.* [25], *Chu and Glover* [26]). A square matrix $Z=[z_{ij}]\in\mathbb{R}^{n\times n}$ is called diagonally dominant matrix if

$$z_{ii}\ge\sum_{j\ne i}^{n}|z_{ij}|,\quad \forall\, i=1,2,\ldots,n.$$

Lemma 1 (*Qiu et al.* [27]). *For any vectors* $\delta(t)\in\mathbb{R}^n$, *two positive integers* κ_0, κ_1, *and matrix* $0<\mathbf{R}\in\mathbb{R}^{n\times n}$, *the following inequality holds*:

$$-(\kappa_1-\kappa_0+1)\sum_{t=\kappa_0}^{\kappa_1}\delta^{\mathrm{T}}(t)\mathbf{R}\delta(t)\le-\left[\sum_{t=\kappa_0}^{\kappa_1}\delta^{\mathrm{T}}(t)\right]\mathbf{R}\left[\sum_{t=\kappa_0}^{\kappa_1}\delta(t)\right].$$